Turbomachinery Coursework

Pusan National University

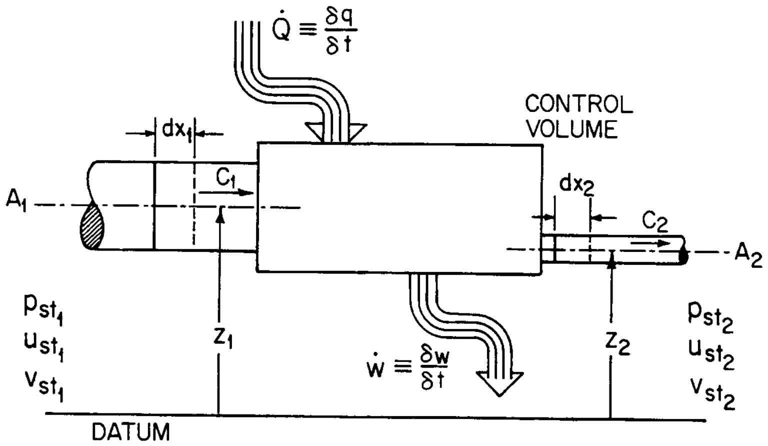
# Gas Turbine
# Performance & Turbine Design



## Thermodynamic and Fluid Dynamic Review

- The first law of thermodynamics
- The steady flow energy equation (SFEE)
- Perfect gas *T-s* and *h-s* diagrams
- Constant-pressure lines on the T-s diagram

## Steady Flow Energy Equation (SFEE)




$$\boxed{\dot{m}(h_2 + \frac{U_2^2}{2} - h_1 - \frac{U_1^2}{2}) = -\dot{W}} \leftarrow \boxed{h = pv + u}$$



## Example: Use SFEE

A turbine passes 20 kg/s of combustion product of known mean specific heat 1130 J/kgK. What is the shaft power output if the mean inlet temperature is 1200°C and mean exhaust temperature is 600°C, both measured with stagnation probes?

From SFEE,

$$\dot{m}(h_{T2} - h_{T1}) = -\dot{W}$$

$$\dot{m}(h_{T2} - h_{T1}) = \dot{m}Cp(T_{T2} - T_{T1})$$

So W=20×1130×(1200-600)=13.56MW

## Perfect Gas *T-s* and *h-s* Diagrams

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Perfect gas law: $$p\upsilon = RT$$

Enthalpy $$h = pv + u$$

Gibbs Equation $$Tds = du + pdv$$

## Constant-Pressure Lines on the T-s Diagram

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Gibbs Equation $$Tds = du + pdv$$

A perfect gas $$p\upsilon = RT$$

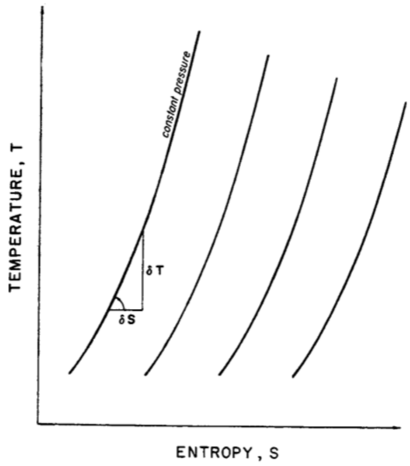
$$pdv + vdp = RdT$$

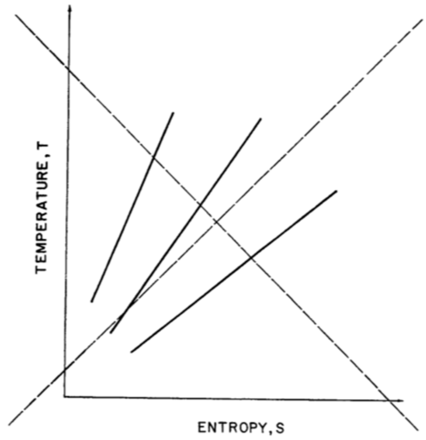
$$du = C_v dT \qquad dh = C_p dT \qquad C_p = C_v + R$$

$$Tds = C_v dT + RdT - vdp = C_p dT - vdp$$

Thus: $$\frac{\delta T}{\delta s}_p = \frac{T}{C_p}$$

## Constant-Pressure Lines on the T-s Diagram



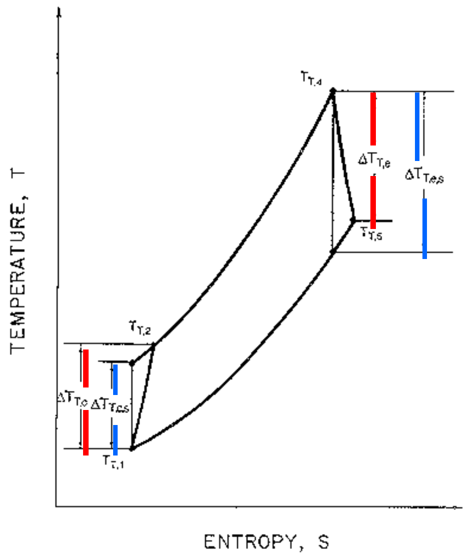




$$\left(\frac{\delta T}{\delta s}\right)_p = \frac{T}{C_p}$$

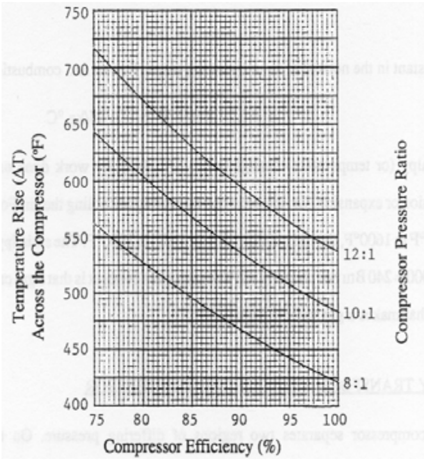


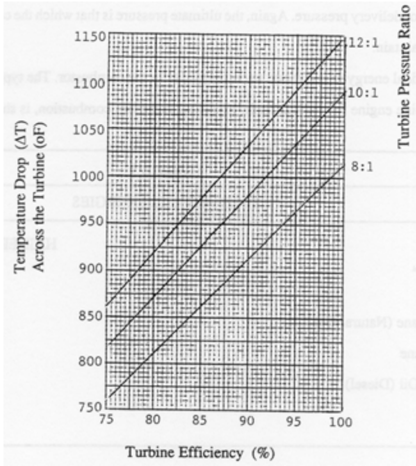
## Efficiency of Turbomachine Components

- The ratio of work transfer between the actual and the ideal processes
- The typical ideal process is: isentropic



## Temperature Rise as a Function of Efficiency





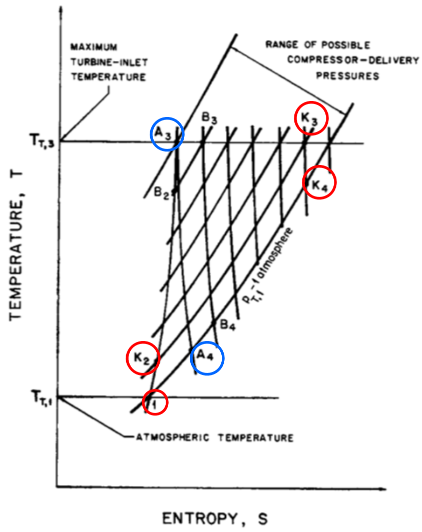

## Gas Turbine Maximum Specific Power

- Low pressure ratio cycle: 1, K2, K3, K4
- High pressure ratio cycle: 1, A3, A4
- Middle range pressure ratios

$$\left(\frac{\dot{W}}{\dot{m}}\right) = -\Delta h_{T,s} = -(h'_{T,2,S} - h_{T,1})$$

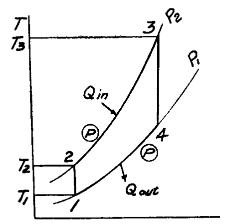
$$= -C_p(T'_{T,2,S} - T_{T,1})$$

$$= -C_p T_{T,1}\left(\left(\frac{p'_{T,2}}{p_{T,1}}\right)^{R/Cp} - 1\right)$$



## Maximum Specific Power of Ideal Joule cycle

$$W_n = W_t - W_C = Cp(T3 - T4) - Cp(T2 - T1)$$
$$= CpT3(1 - \frac{T4}{T3}) - CpT1(\frac{T2}{T1} - 1)$$
$$= CpT3(1 - \frac{1}{\left(\frac{P3}{P4}\right)^{(\gamma-1)/\gamma}}) - CpT1(\left(\frac{P2}{P1}\right)^{(\gamma-1)/\gamma} - 1)$$
$$= CpT3\left(\frac{P_r^{(\gamma-1)/\gamma} - 1}{P_r^{(\gamma-1)/\gamma}}\right) - CpT1\left(P_r^{(\gamma-1)/\gamma} - 1\right)$$




$$\frac{P_3}{P_4} = \frac{P_2}{P_1} = P_r$$

For giving limiting temperature T3 and fixed T1

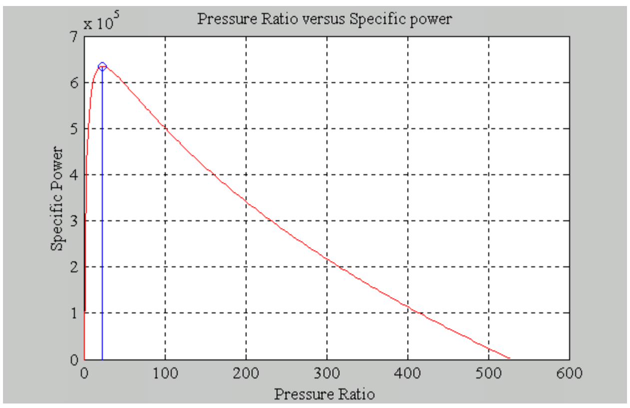
$$\frac{dW_n}{d(P_r^{(\gamma-1)/\gamma})} = CpT3\left(\frac{1}{P_r^{(\gamma-1)/\gamma}}\right)^2 - CpT1 = 0$$

Thus $\boxed{P_r^{(\gamma-1)/\gamma} = \sqrt{\frac{T3}{T1}}}$ or $\boxed{P_r = \left(\frac{T3}{T1}\right)^{\gamma/(2(\gamma-1))}}$

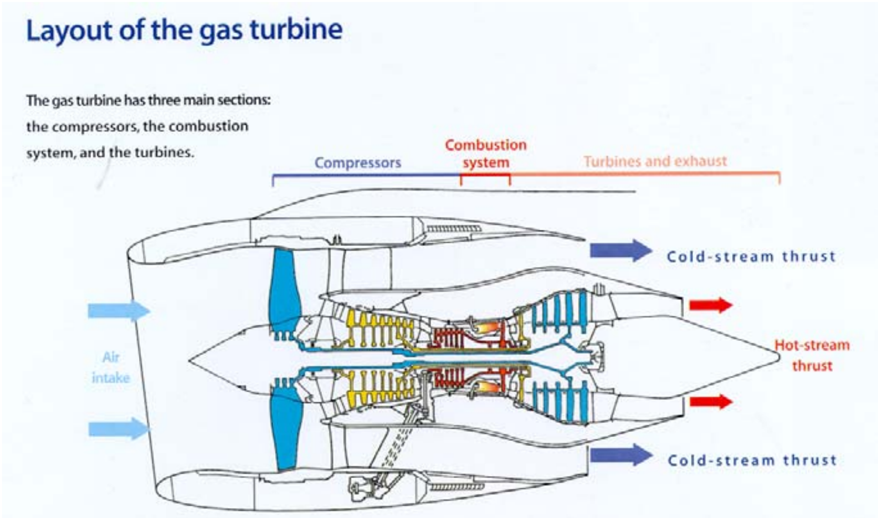


## Gas Turbine Maximum Specific Power

- There is an optimum pressure ratio at which maximum net power is produced.
- T3=1800K
- T1=300K

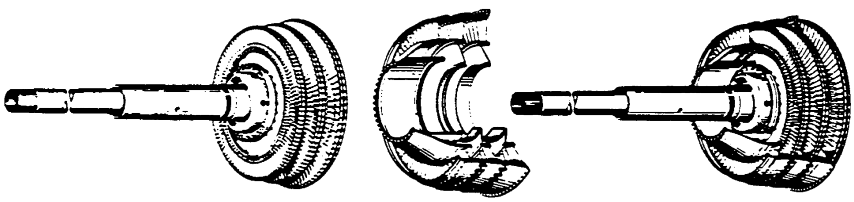


# Gas Turbine





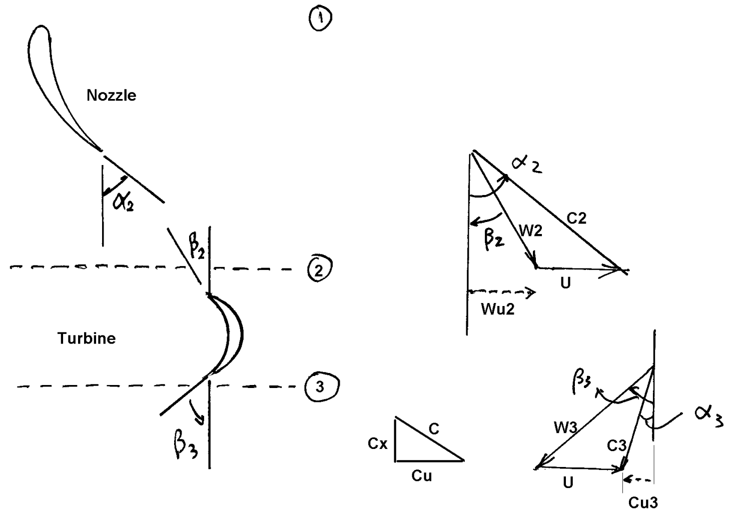

# Gas Turbine



(a) Rotor (b) Stator (c) Combined Assembly

## Turbine Velocity Diagram

Normal stage with a constant $C_x$.






## 50% Reaction Turbine

- Euler equation $$h_{T1} - h_{T3} = u\left(C_{u2} - C_{u3}\right)$$

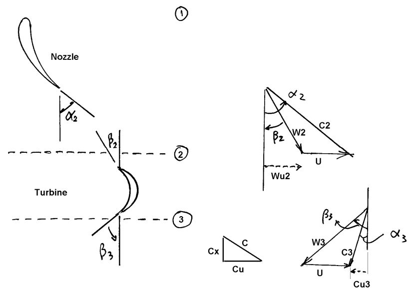
$$R = \frac{h_2 - h_3}{h_{T1} - h_{T3}} = \frac{1/2\left(W_3^2 - W_2^2\right)}{u\left(C_{u2} - C_{u3}\right)} = \frac{1/2\left((W_{u3}^2 + C_x^2) - (W_{u2}^2 + C_x^2)\right)}{u\left(u + W_{u2} + |C_{u3}|\right)}$$

$$= \frac{1/2\left((|C_{u3}| + u)^2 - (W_{u2}^2)\right)}{u\left(u + W_{u2} + |C_{u3}|\right)} = \frac{1}{2}\frac{\left((|C_{u3}| + u) - (W_{u2})\right)}{u} = \frac{1}{2}\frac{C_x \tan\beta_3 - C_x \tan\beta_2}{u}$$

$$R = \frac{1}{2}\frac{C_x}{u}\left(\tan\beta_3 - \tan\beta_2\right) \qquad \tan\beta_2 = \tan\alpha_2 - \frac{u}{C_x}$$

$$R = \frac{1}{2} + \frac{1}{2}\frac{C_x}{u}\left(\tan\beta_3 - \tan\alpha_2\right)$$

$$\boxed{\beta_3 = \alpha_2}$$



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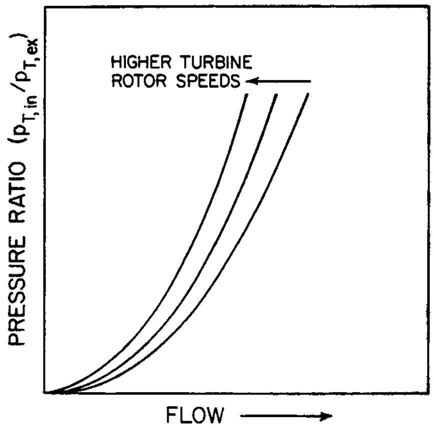
## Turbine Characteristics

- Lossless incompressible flow through a nozzle. The SFEE gives

$$P_T - P_{st} = \frac{\rho C^2}{2}$$

- Mass flow rate is

$$m = \rho c A = \left(2 \times \left(P_T - P_{st}\right) / \rho\right)^{0.5} \rho A$$
$$= \left(2 \times \left(P_T - P_{st}\right) \times \rho\right)^{0.5} A$$

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## Design an Axial Flow Turbine

- Turbine shape,
- Size
- Number of blades

## Preliminary Turbine Design

- 1. Obtain a set of turbine specifications, including at least.
  - The fluid to be used
  - The fluid total temperature and pressure at inlet
  - The fluid total or static pressure at outlet
  - Either the fluid mass flow rate or the turbine power output required
  - And perhaps the desired shaft speed.
- 2. Choose the velocity diagram at the mean diameter.



## Preliminary Turbine Design

- 3. The velocity diagrams at other radial locations are next chosen using radial equilibrium
- 4. Choose the number of blades for each blade row and their shape.
- 5. Predict turbine performance at design and off design conditions.
- 6. Stress calculation for the blades so produced.

## Preliminary Turbine Design

- 7. Heat transfer analysis for high temperature turbines.
- 8. Combine the mechanical and thermal stresses. Check against various criteria of failure.
- 9. Check natural frequencies of the blades for the full speed range.

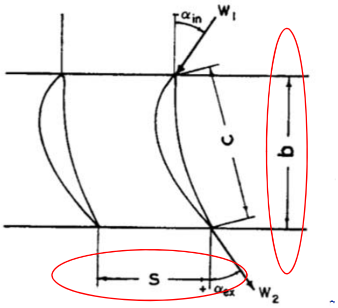


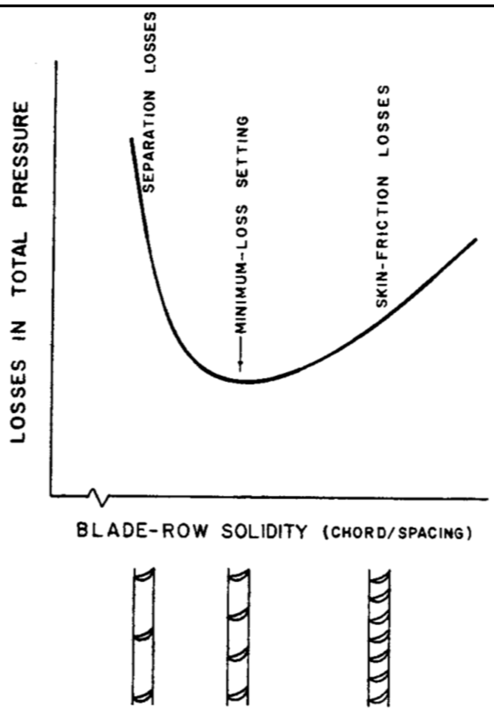
## Blade Shape, Spacing and Number

- Calculation of solidity for design-point operation.
- Estimate “induced incidence” correction
- Estimate out flow deviation from correlations
- Choice of leading and trailing edge radii
- Find the optimum setting angle.
- Selection of number of blades from consideration of performance, vibration, and heat transfer analysis.

## Solidity

- A small solidity means small number of widely spaced blades.
- A high solidity means the blades are close packed.






## Optimum solidity

- Is related to the aerodynamic loading of the blade.
- Zweifel (1945) found that minimum loss solidities could be well correlated by setting the tangential lift coefficient, $C_L$, at a constant value of 0.8,
- Current design practice uses lift coefficients between 0.9 and 1.0, and occasionally higher

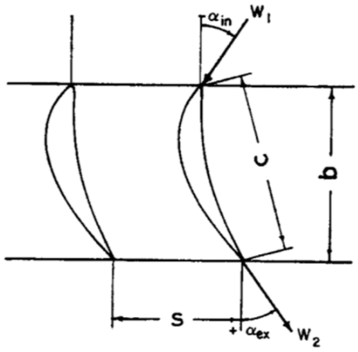
## Tangential Lift Coefficient

- Tangential force F is

$$F = m(w_{in}\sin\alpha_{in} - w_{ex}\sin\alpha_{ex})$$

Mass flow

$$m = s \times h \times \rho_{ex,st} \times (w_{ex} \times \cos\alpha_{ex})$$




$$\boxed{C_L = \frac{F}{b \times h}\frac{1}{w_{ex}^2\rho/2} = 2\left(\frac{s}{b}\right)\cos^2\alpha_{ex}\left[\left(\frac{w_{in}\sin\alpha_{in}}{w_{ex}\cos\alpha_{ex}}\right) - \tan\alpha_{ex}\right]}$$



## Lift Coefficient for Constant Axial Velocity

Axial velocity $\quad C_Z = w_{in}\cos\alpha_{in} = w_{ex}\cos\alpha_{ex}$

Lift Coefficient $\quad C_L = 2\left(\frac{s}{b}\right)\cos^2\alpha_{ex}(\tan\alpha_{in} - \tan\alpha_{ex})$

- Axial solidity for the optimum lift coefficient of 0.8

$$\boxed{\left(\frac{b}{s}\right)_{op} = \left|2.5\cos^2\alpha_{ex}(\tan\alpha_{in} - \tan\alpha_{ex})\right|}$$

## Number of Blades

Axial turbines have been built with:

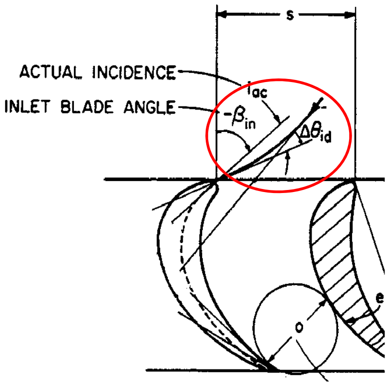
- Wind turbines: 1 to 24 blades
- Water turbines: 3 to 30 blades
- Gas turbines (include steam turbines): 11 to 110 blades



## Induced Incidence

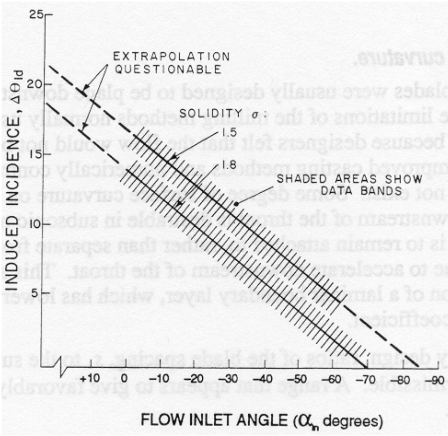
- Caused by the circulation around the blades
- The correction of induced incidence

$$\Delta\theta_{id} = 14\left(1 - \frac{|\alpha_{in}|}{70°}\right) + 9\left(1.8 - \frac{c}{s}\right)$$



# The Correction of Induced Incidence

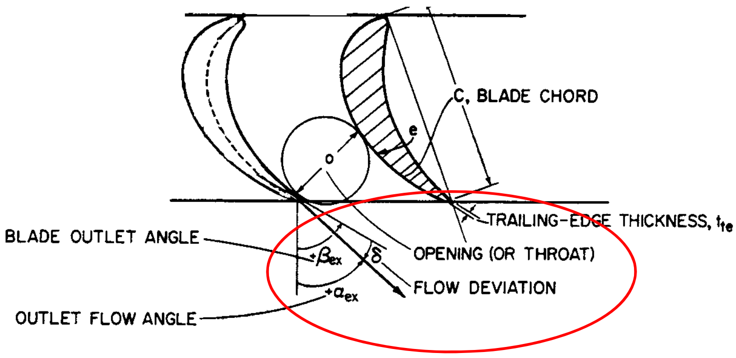
$$\Delta\theta_{id} = 14\times\left(1-\frac{|\alpha_{in}|}{70°}\right)+9\times\left(1.8-\frac{c}{s}\right)$$






# Deviation

- The deviation is of critical importance to the turbine designer



# Deviation

- Ainley and Mathieson (1951) published one correlation method.

$$\left|\alpha_{ex}\right|_{0<Ma<0.5} = \frac{7}{6}\left(\left|\cos^{-1}\left(\frac{o}{s}\right)\right| - 10^\circ\right) + 4^\circ\left(\frac{s}{e}\right)$$

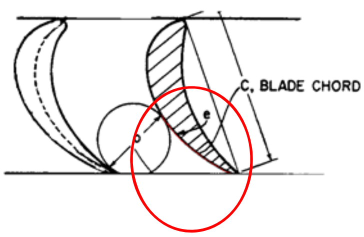
$$\left|\alpha_{ex}\right|_{Ma=1.0} = \left|\cos^{-1}\left(\frac{o}{s}\right)\right| - \left(\frac{s}{e}\right)^{1.786+4.128(s/e)}$$

$$\left|\alpha_{ex}\right|_{0.5<Ma<1.0} = \left|\alpha_{ex}\right|_{0<Ma<0.5} - (2Ma-1)\left(\left|\alpha_{ex}\right|_{0<Ma<0.5} - \left|\alpha_{ex}\right|_{Ma=1.0}\right)$$



# Surface Curvature After Throat

- In the past, it was designed to be plane.
- Now, it is designed with surface curvature
- The ratio of blade spacing, s, over the surface curvature, e, between 0.25 and 0.625, was found to be favourable.



# Blade Leading Edge

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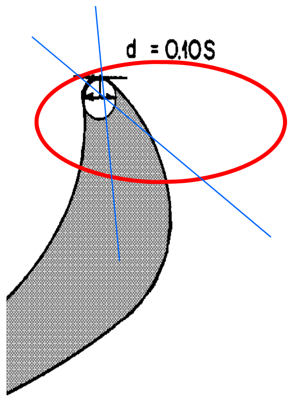
- Usually the leading edge radius falls between 0.05s and 0.1s
- The angle between the tangents made by the two blade surfaces to the leading edge circle need to be specified.




# Trailing Edges

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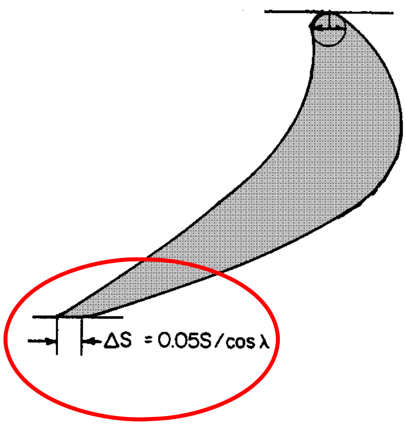
- The thickness is usually between 0.015c and 0.05c where c is the chord.



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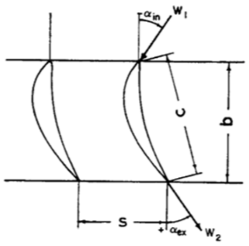
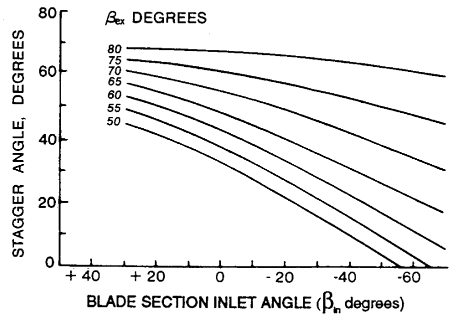
## Setting Angle

The setting angle, λ, is related to the axial chord, b, and the blade chord, c, by

$$\boxed{\cos\lambda = \frac{b}{c}}$$




A guide for obtaining the setting angle by Kacker and Okapuu 1981

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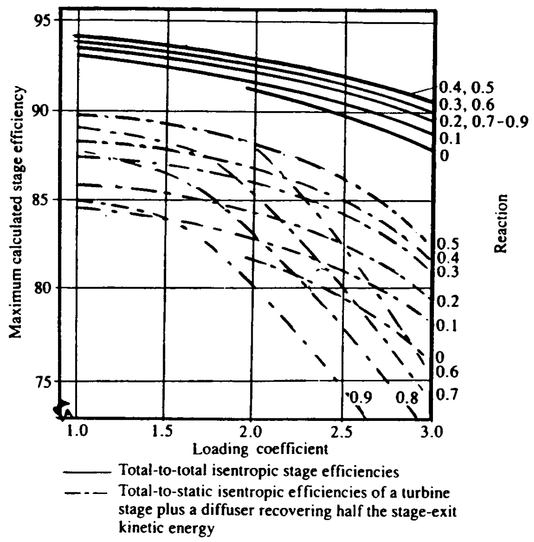
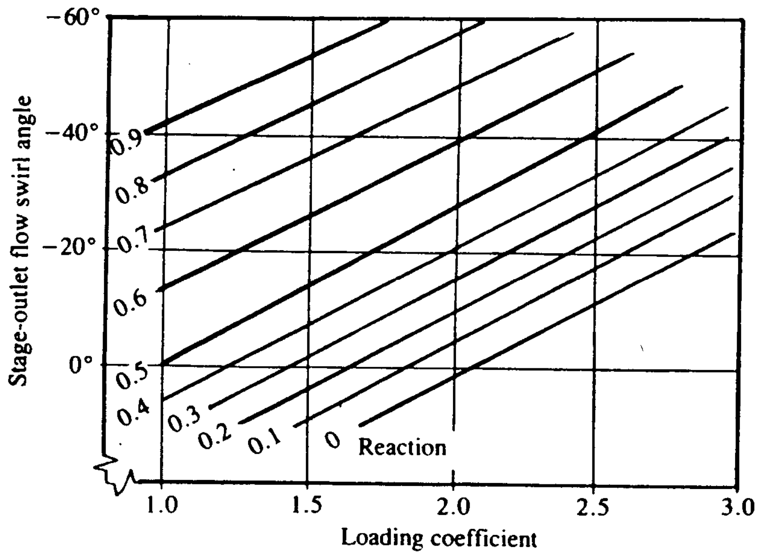
## Why 50% Reaction?

- The 50% reaction diagrams have the highest efficiency

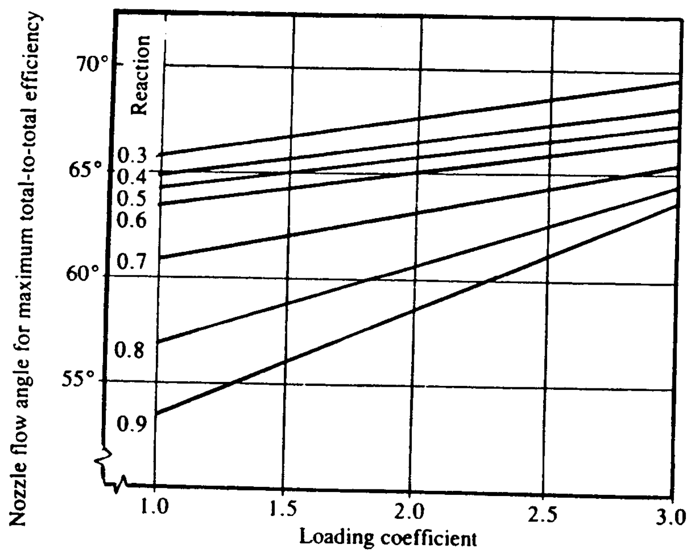


## Effects of Loading on Rotor-Outlet Flow Swirl






## Optimum Nozzle Flow Angle Versus Loading Coefficient and Reaction



## Example: Design a Turbine Blade!

Knowns:

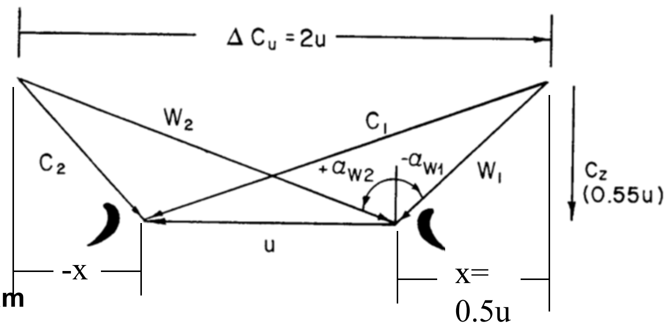
- Reaction: 50%;
- Work coefficient: 2.0;
- Flow coefficient: 0.55;
- Throat Mach number: 0.75;
- Leading edge diameter: 1/10 of spacing;
- Trailing edge thickness: 0.02c;
- The ratio of the spacing to the surface curvature downstream of the throat: 0.333.



## Step 1/4: Find Velocity Diagram

$$R50 = 1 - \frac{C_{u,2} + C_{u,1}}{2u}$$

$$\psi = -\frac{\Delta(C_u)}{u} = -\frac{(-x)-(u+x)}{u} = 2$$




- **Velocity diagram**
- 
- 

$$\tan\alpha_{w1} = \frac{-0.5u}{0.55u}$$

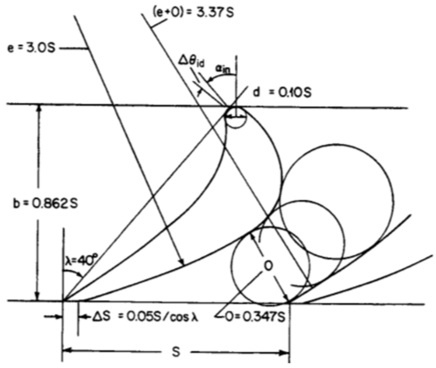
$$\tan\alpha_{w2} = \frac{1.5u}{0.55u}$$

$\alpha_{in}=\alpha_{w1}=-42.27°$

$\alpha_{ex}=\alpha_{w2}=69.86°$

## Step 2a/4: Find Optimum Solidity

- Use optimum lift coefficient of 1.0




$$\left(\frac{b}{s}\right)_{op} = \left|2.0\cos^2\alpha_{ex}\left(\tan\alpha_{in} - \tan\alpha_{ex}\right)\right| = 0.862$$



## Step 2b/4: Throat Opening

$$\left|\alpha_{ex}\right|_{0<Ma<0.5} = \frac{7}{6}\left(\left|\cos^{-1}\left(\frac{o}{s}\right)\right| - 10^\circ\right) + 4^\circ\left(\frac{s}{e}\right)$$

$$\left|\alpha_{ex}\right|_{Ma=1.0} = \left|\cos^{-1}\left(\frac{o}{s}\right)\right| - \left(\frac{s}{e}\right)^{1.786+4.128(s/e)}$$

Predicted outlet flow angle

| | | |
|---|---|---|
| $Cos^{-1}(o/s)$ | 69° | 70° |
| $\alpha_{ex,Ma=0.5}$ | 70.167° | 71.333° |
| $\alpha_{ex,Ma=1.0}$ | 68.971° | 69.971° |
| $\alpha_{ex,Ma=0.75}$ | 69.569° | 70.652° |

By interpolating, it is found $\cos^{-1}(o/s)$=69.71° is needed to obtain the desired outlet angle, 69.86°.

$$\frac{o}{s} = 0.3468$$

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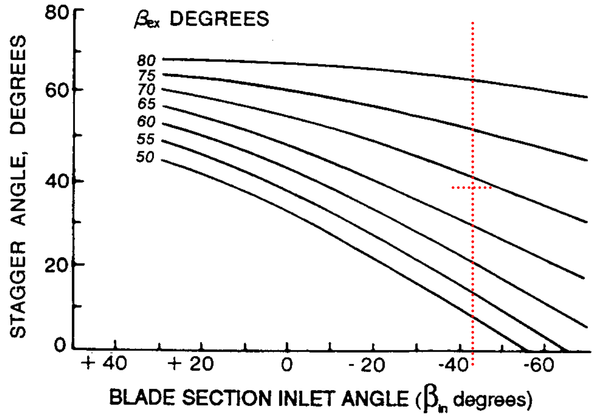
# Step 3/4: Find Setting Angle

- By looking at right figure (Kacker et al. 1981), the first guess of angle can be 40°.

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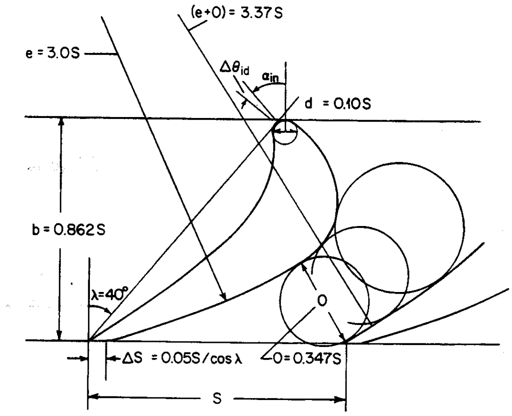
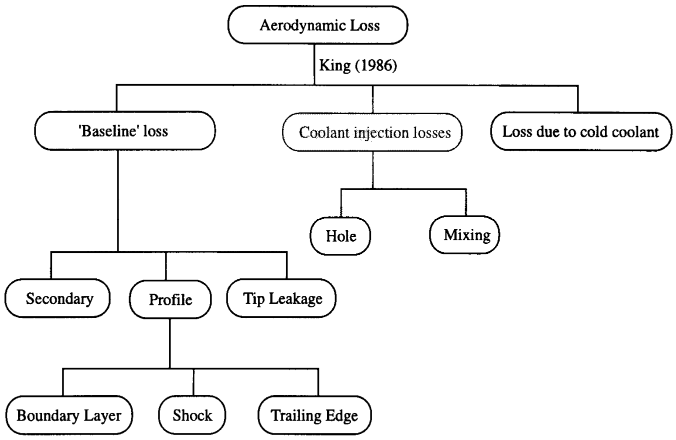
# Step 4/4: Draw the Blade Profile

- Choose the blade inlet angle in the range between $\alpha_{in}$ and $\alpha_{in}+\Delta\theta_{id}$.
- Connect the leading edge to the throat using smooth curves.
- Acceptable passages are those giving continuous reductions of area up to the throat.



# Turbine Efficiency

Loss = 1 - Efficiency



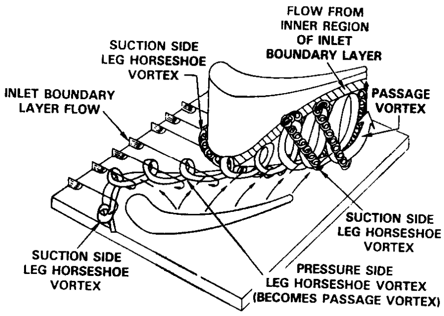


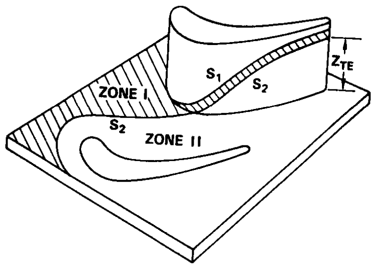

# Profile Loss

- Loss that would appear in a two-dimensional cascade of infinite span.
- It is made up of
  - boundary layer loss
  - shock loss
  - trailing edge losses.

# Secondary Loss

- A complex product of the interaction between endwall boundary and mainstream flow



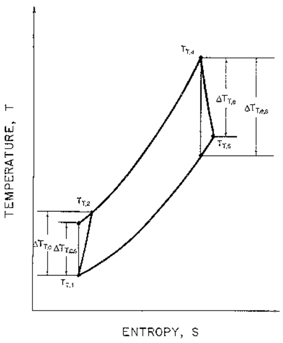


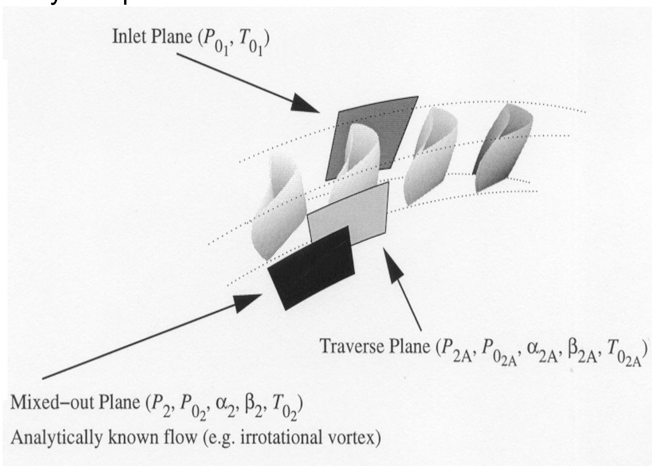

# Other Losses

- **Tip leakage loss** is found in un-shrouded blade rows, usually rotors, where the pressure difference from pressure surface to suction side drives flow through any gaps between the blade and endwall.
- **Loss caused by coolant injection**: loss inside the cooling hole plus the loss due to coolant mixing with mainstream.

## Measure Turbine Efficiency

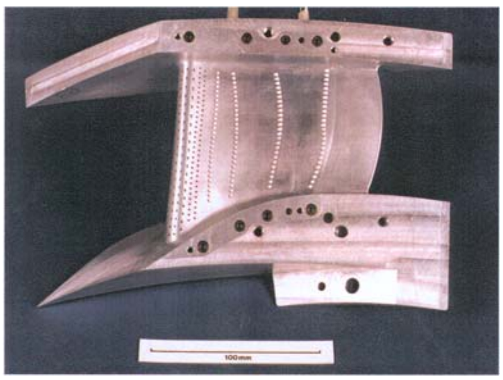
- Downstream velocity and pressure traverse



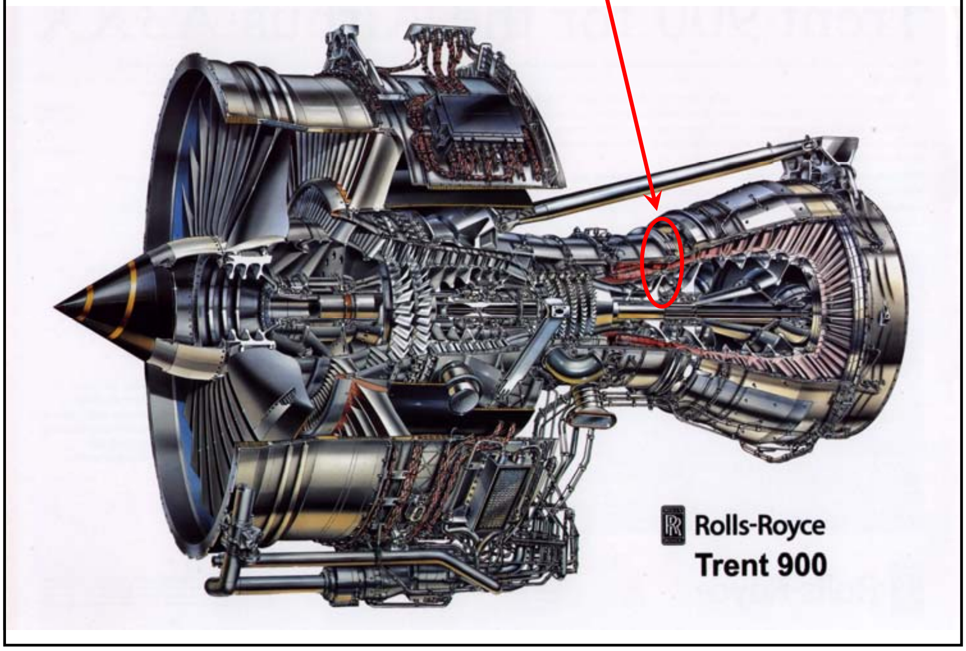



## A Turbine Nozzle Guide Vane



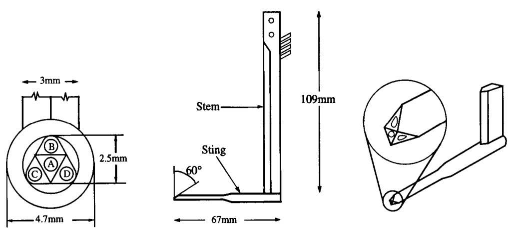
# First Stage NGV

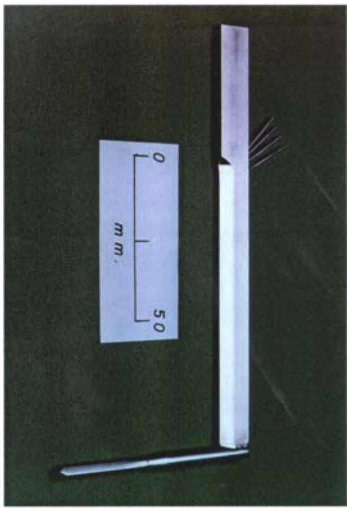







# Four Hole Probe for Downstream Traverse



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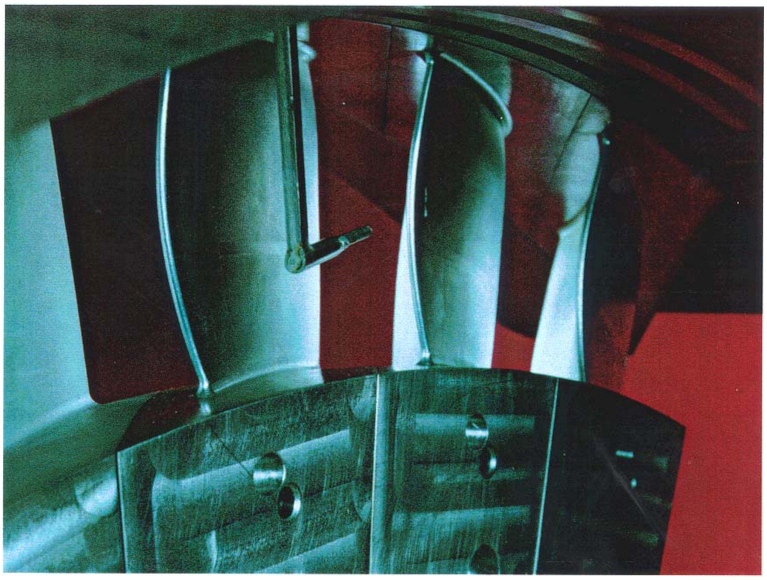
## Downstream Traverse: Four Hole Probe in Position



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## Typical Downstream Traverse Results

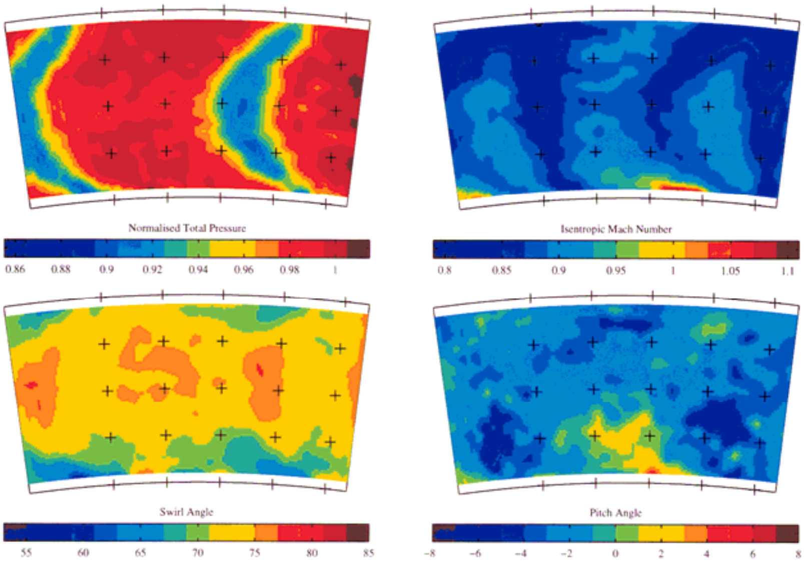


**Figure 6.1:** *Traverse maps of flowfield variables – no cooling (Case "A").*

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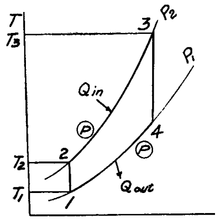
### Maximum Specific Power of Ideal Joule cycle: The reason for cooling

$$W_n = W_t - W_C = Cp(T3 - T4) - Cp(T2 - T1)$$
$$= CpT3(1 - \frac{T4}{T3}) - CpT1(\frac{T2}{T1} - 1)$$
$$= CpT3(1 - \frac{1}{\left(\frac{P3}{P4}\right)^{(\gamma-1)/\gamma}}) - CpT1(\left(\frac{P2}{P1}\right)^{(\gamma-1)/\gamma} - 1)$$
$$= CpT3\left(\frac{P_r^{(\gamma-1)/\gamma} - 1}{P_r^{(\gamma-1)/\gamma}}\right) - CpT1\left(P_r^{(\gamma-1)/\gamma} - 1\right)$$

$$\frac{P_3}{P_4} = \frac{P_2}{P_1} = P_r$$




For giving limiting temperature T3 and T1

$$\frac{dW_n}{d(P_r^{(\gamma-1)/\gamma})} = CpT3\left(\frac{1}{P_r^{(\gamma-1)/\gamma}}\right)^2 - CpT1 = 0$$

Thus $P_r^{(\gamma-1)/\gamma} = \sqrt{\frac{T3}{T1}}$ or $P_r = \left(\frac{T3}{T1}\right)^{\gamma/(2(\gamma-1))}$

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# Turbine Cooling

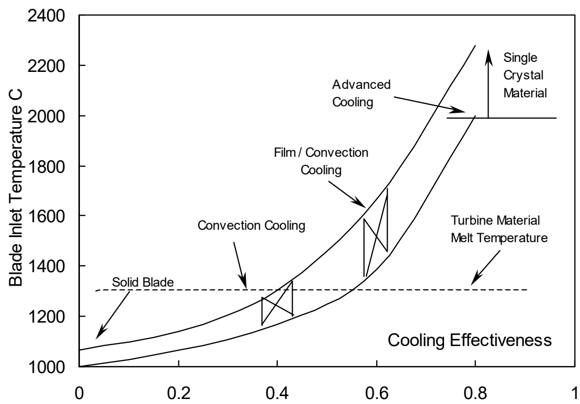
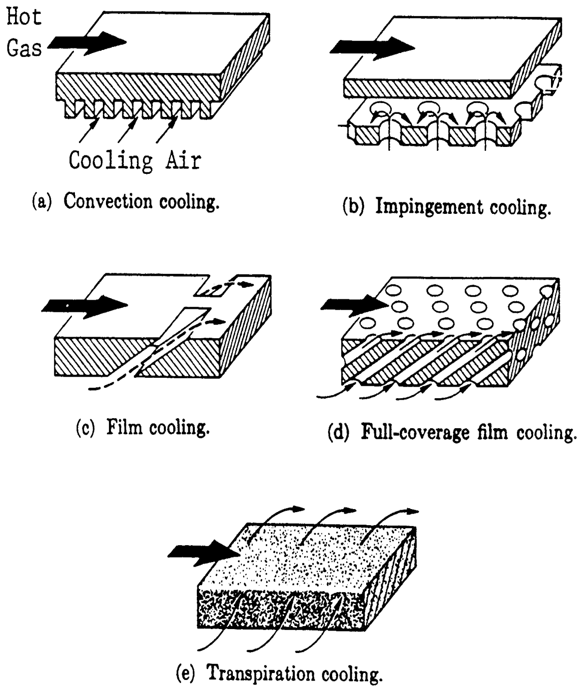



Figure 3-18: Turbine blade inlet temperature versus cooling technology

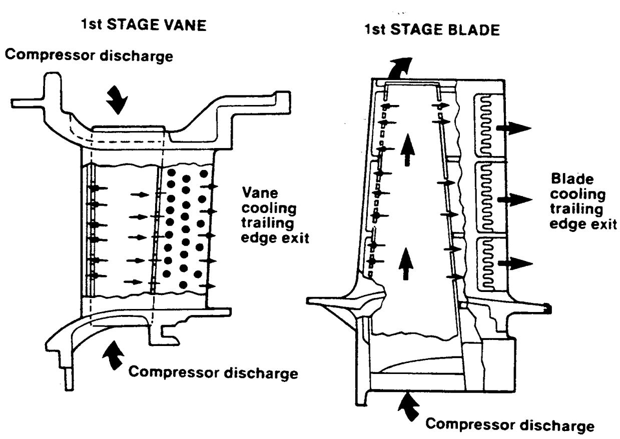
## Turbine Blade Cooling



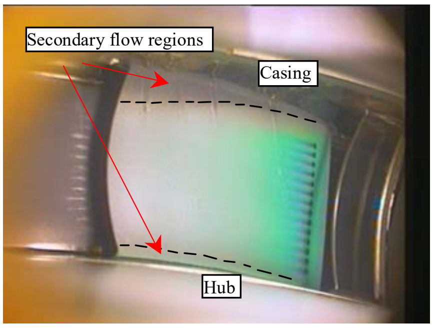
(a) Convection cooling.
(b) Impingement cooling.
(c) Film cooling.
(d) Full-coverage film cooling.
(e) Transpiration cooling.

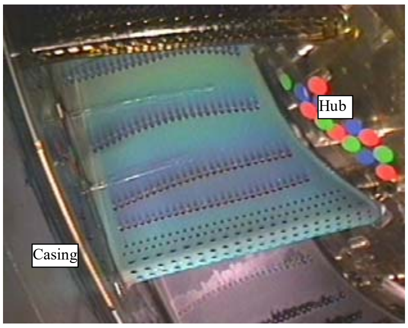


## Turbine Film Cooling



## Film Cooling Study: Transient Method






## Summary

- Turbine velocity diagram
- Turbine design
- Turbine efficiency and cooling