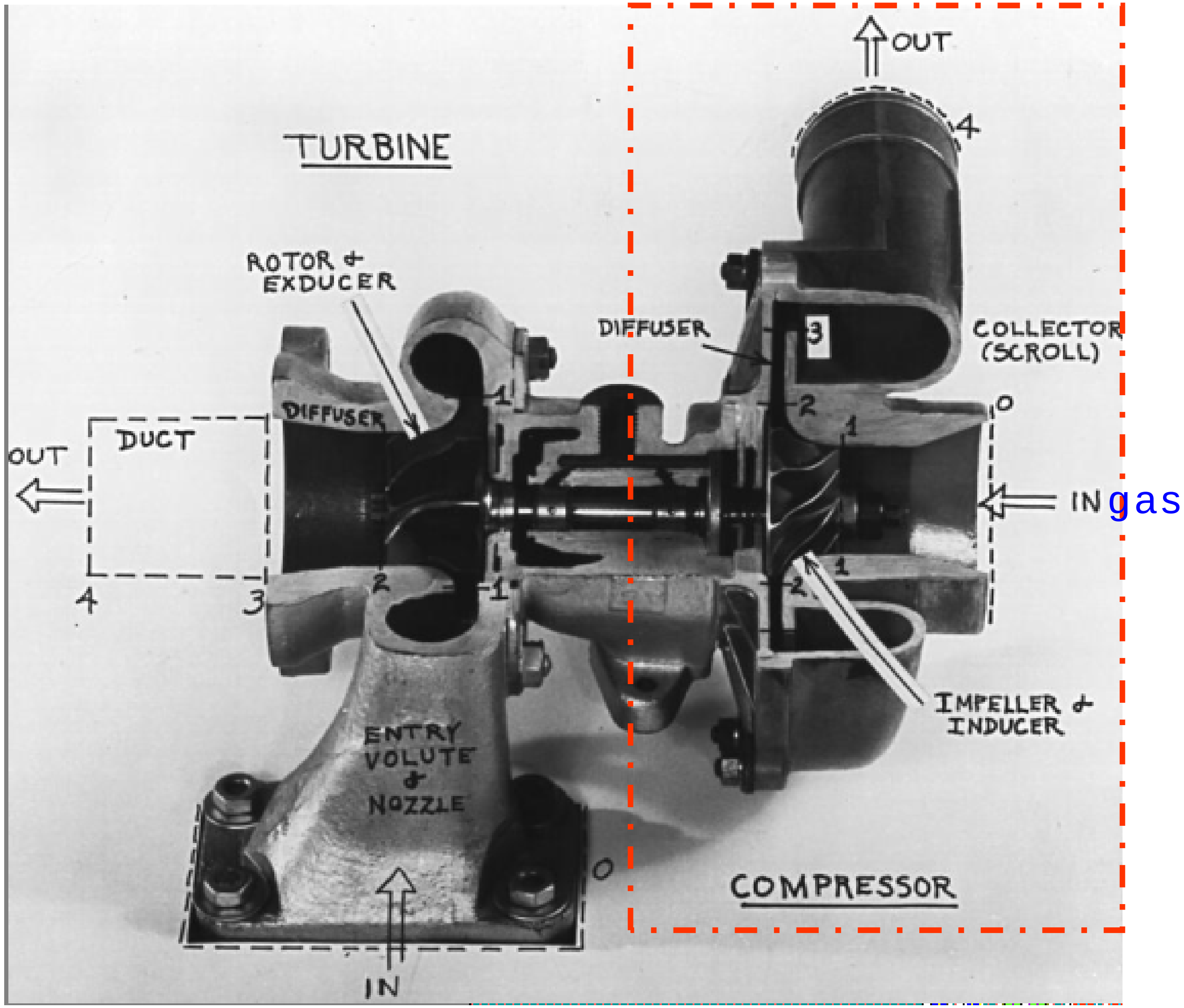
Centrifugal Compressor



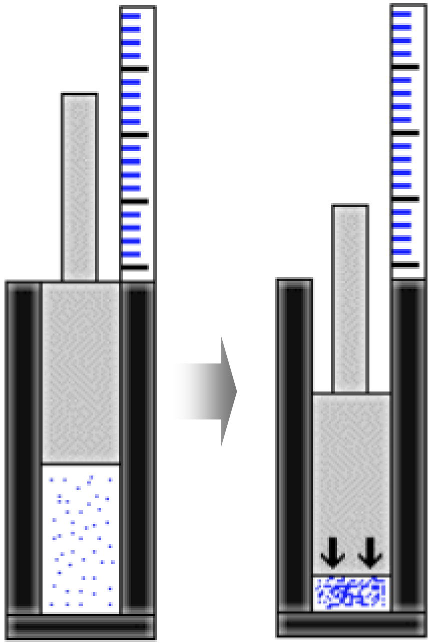
P_{01}: total pressure

T_{01}: total temperature

N: rotational speed

$\dot{m}$: mass flow rate

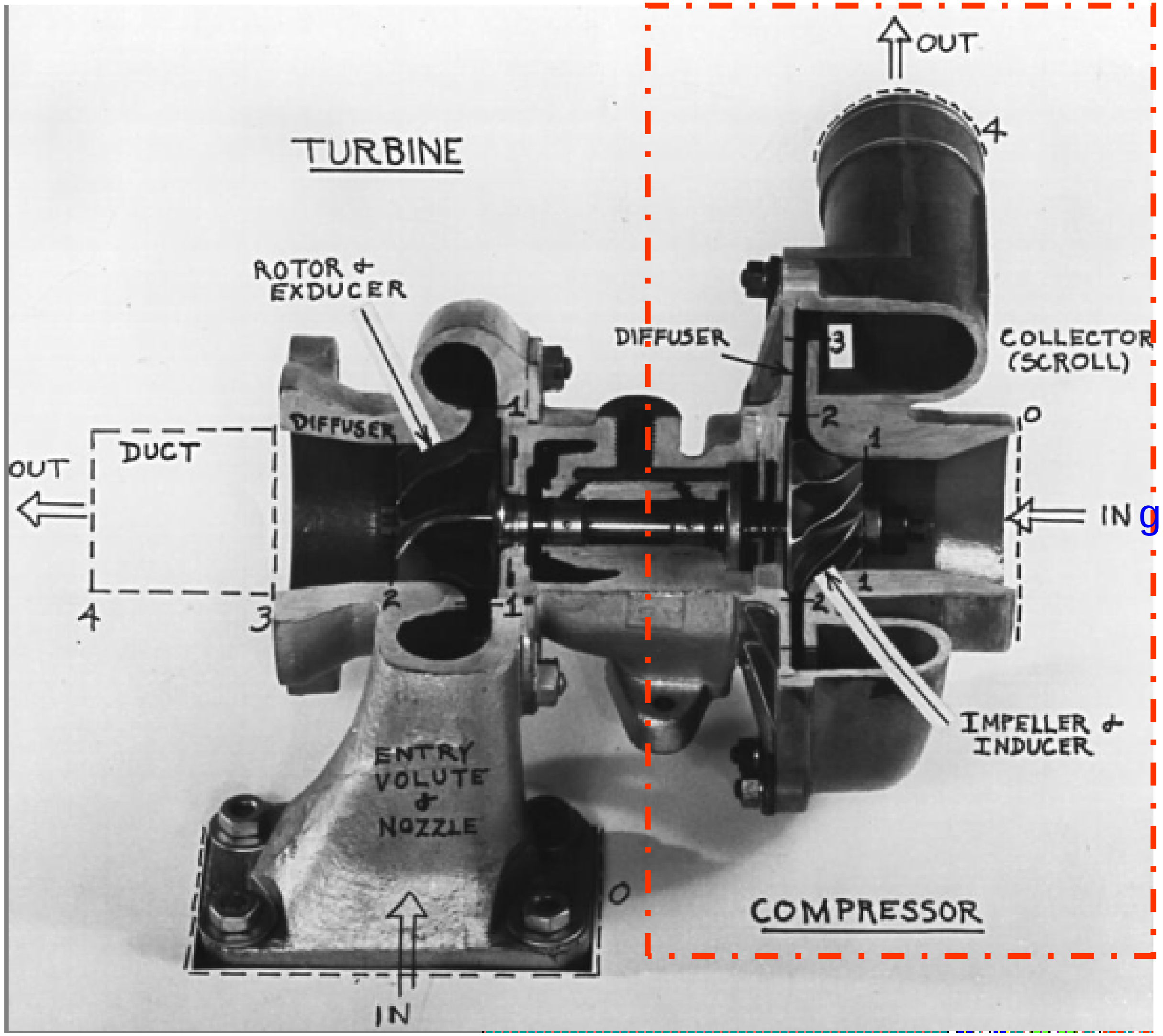
Working Mechanism



- The basic equations developed for the pump apply to the compressors with the difference that density does increase
- The thermodynamic equation of state of a perfect gas must be considered in the detailed calculation(density, pressure, temp.)
- Main difference in carrying out a compressor analysis, as opposed to a pump analysis, is the appearance of an enthalpy term in place of the flow work or pressure-head term
- A single stage of a centrifugal compressor can produce a pressure ratio of 5 times that of a single stage of an axial flow compressor

Based on an ideal gas law of $Pv = \rho RT$,
compression provides increase in pressure

Centrifugal Compressor



gas

P_{01}: total pressure

T_{01}: total temperature

N: rotational speed

$\dot{m}$: mass flow rate

Centrifugal Compressor

Total Pressure & Temperature

P_{01}: total pressure

$$P_o = P_s + P_d + P_e$$

T_{01}: total temperature

$$T_o = T_s(1 + 0.2M^2)$$

SAT degrees Celsius	Indicated Mach Number							
	0.50	0.60	0.70	0.75	0.80	0.85	0.90	0.95
	Total Air Temperature (TAT) degrees Celsius							
-75	-65	-61	-56	-53	-50	-46	-43	-39
-74	-64	-60	-54	-52	-49	-45	-42	-38
-73	-63	-59	-53	-50	-47	-44	-41	-37
-72	-62	-58	-52	-49	-46	-43	-39	-36
-71	-61	-56	-51	-48	-45	-42	-38	-35
-70	-60	-55	-50	-47	-44	-41	-37	-33
-69	-59	-54	-49	-46	-43	-40	-36	-32
-68	-58	-53	-48	-45	-42	-38	-35	-31
-67	-57	-52	-47	-44	-41	-37	-34	-30
-66	-56	-51	-46	-43	-39	-36	-32	-29
-64	-54	-49	-44	-40	-37	-34	-30	-26
-62	-51	-47	-41	-38	-35	-31	-28	-24
-60	-49	-45	-39	-36	-33	-29	-25	-22
-58	-47	-43	-37	-34	-30	-27	-23	-19

Centrifugal Compressor

- Thermodynamic Relations

Mach number

$$Ma = \frac{V}{a} = \frac{ND}{\sqrt{h_1(\gamma - 1)}},$$

$$a = \sqrt{\gamma RT} = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{(\gamma - 1)h}$$

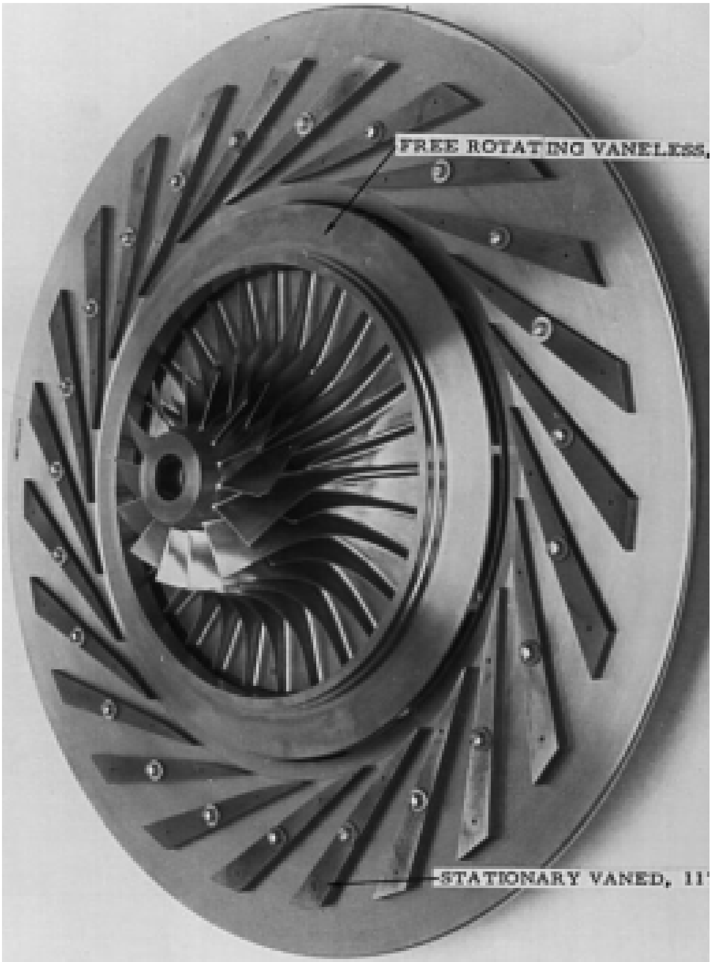
$$h = C_p T, \quad \frac{p}{\rho} = RT, \quad \frac{R}{C_p} = \frac{\gamma - 1}{\gamma}$$

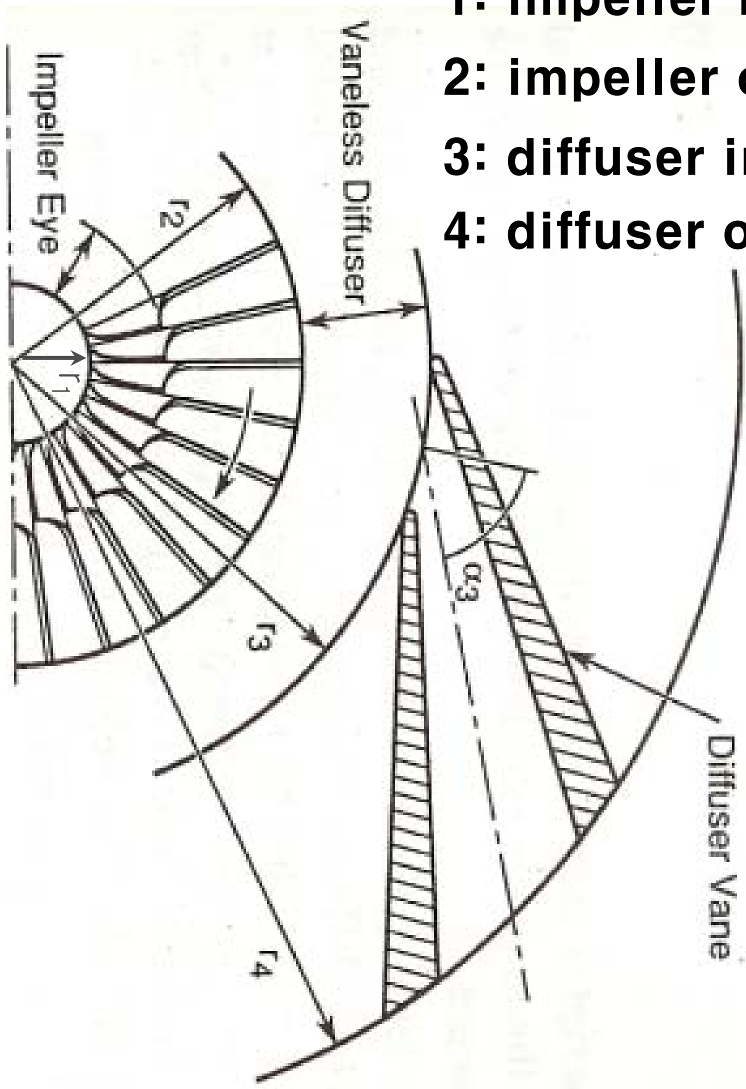
T_{01}: total temperature

$$T_o = T_s(1 + 0.2M^2)$$

NEXTMEMS

Thermodynamic Process





1: impeller inlet

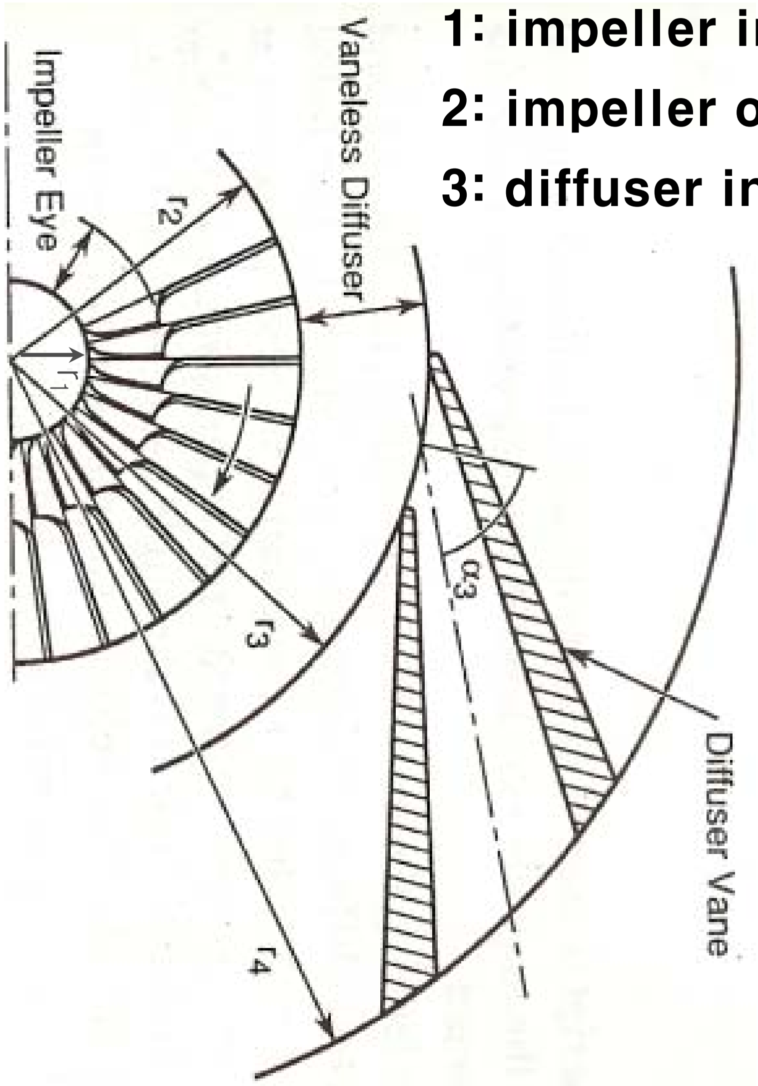
2: impeller outlet

3: diffuser inlet

4: diffuser outlet

NEXTMEMS

Enthalpy-Entropy Diagram

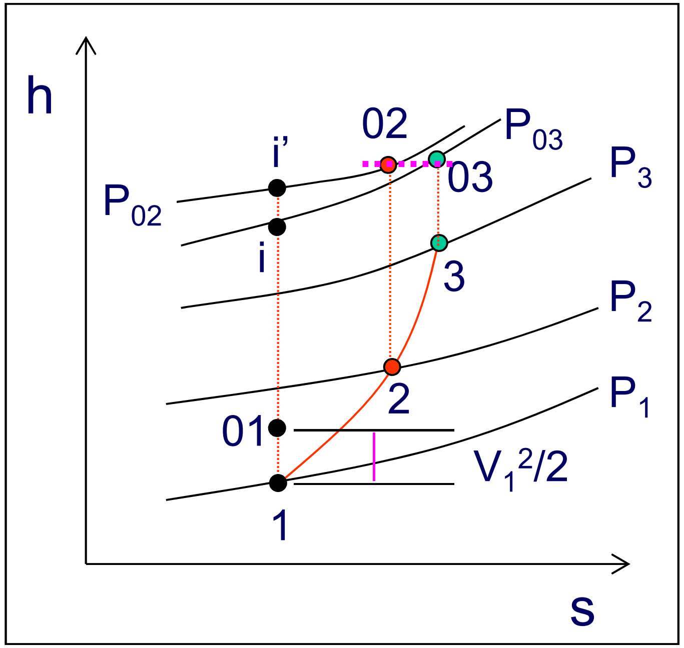


1: impeller inlet

2: impeller outlet

3: diffuser inlet

$$h + \frac{V^2}{2} = h_o$$



- note
  - static P vs. total Po
  - static h vs. total ho

Centrifugal Compressor

Thermodynamic Relations

Isentropic Process $(\Delta s = 0) \equiv$ Adiabatic and Reversible

1. Pure Substance (H_2O, R134a):

$$\Delta s = 0 \Rightarrow \boxed{s_1 = s_2 \Rightarrow \text{use tables}}$$

2. Incompressible Liquids (or Solids):

$$\Delta s = C_{liq} \cdot \ln\left(\frac{T_2}{T_1}\right) = 0 \Rightarrow \boxed{T_1 = T_2}$$

3. Ideal Gas:

$$\Delta s = C_v \cdot \ln\left(\frac{T_2}{T_1}\right) + R \cdot \ln\left(\frac{v_2}{v_1}\right) = C_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{P_2}{P_1}\right) = 0$$

thus: $\ln\left(\frac{T_2}{T_1}\right) = -\left(\frac{R}{C_v}\right) \cdot \ln\left(\frac{v_2}{v_1}\right) = \left(\frac{R}{C_v}\right) \cdot \ln\left(\frac{v_1}{v_2}\right) = \left(\frac{R}{C_p}\right) \cdot \ln\left(\frac{P_2}{P_1}\right)$

also: $C_p - C_v = R$, $\left(\frac{C_p}{C_v}\right) = k \Rightarrow \left(\frac{R}{C_v}\right) = k - 1$, $\left(\frac{R}{C_p}\right) = \frac{k-1}{k}$

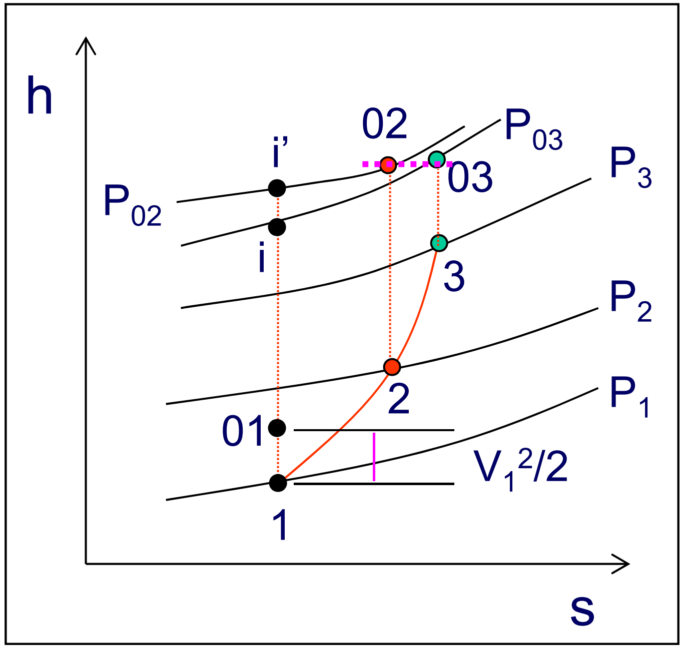
thus finally:

$$\boxed{\begin{array}{lll}
\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{k-1} & \Leftrightarrow & T \cdot v^{k-1} = \text{constant} \\
\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{P_1}{P_2}\right)^{\frac{1-k}{k}} & \Leftrightarrow & T \cdot P^{\frac{1-k}{k}} = \text{constant} \\
\left(\frac{P_2}{P_1}\right) = \left(\frac{v_1}{v_2}\right)^{k} & \Leftrightarrow & P \cdot v^{k} = \text{constant}
\end{array}}$$

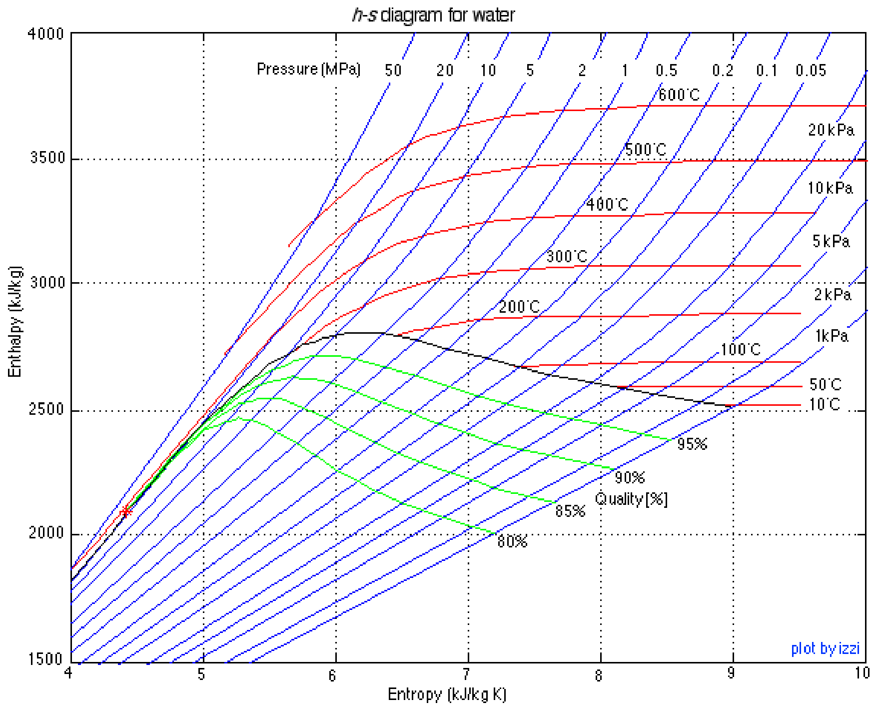
NEXTMEMS

Enthalpy-Entropy Diagram

Pressure



Temperature



Transferred Energy

- Energy transfer

$$\left(h_1 + \frac{V_1^2}{2}\right) = \left(h_2 + \frac{V_2^2}{2}\right) + E$$

$\longrightarrow$ $h_{01} = h_{02} + E$

$$E = \eta_m \left(h_{o3} - h_{o1}\right) \quad \text{eq1}$$

h_o : total enthalpy
h : static enthalpy

or

$$E = U_2 V_{u2}^{'} \quad \text{eq2}$$

η_m : mechanical efficiency accounts for frictional losses occurring between moving mechanical parts, which are typically bearings, seals and disk friction

$$\eta_m = \frac{\left(\dot{m} + \dot{m}_L\right) g H_{in}}{P} \quad \text{in pump}$$

- noting that units in two eqns must be taken a care in calculation

NEXT MEMS

Useful Energy Input

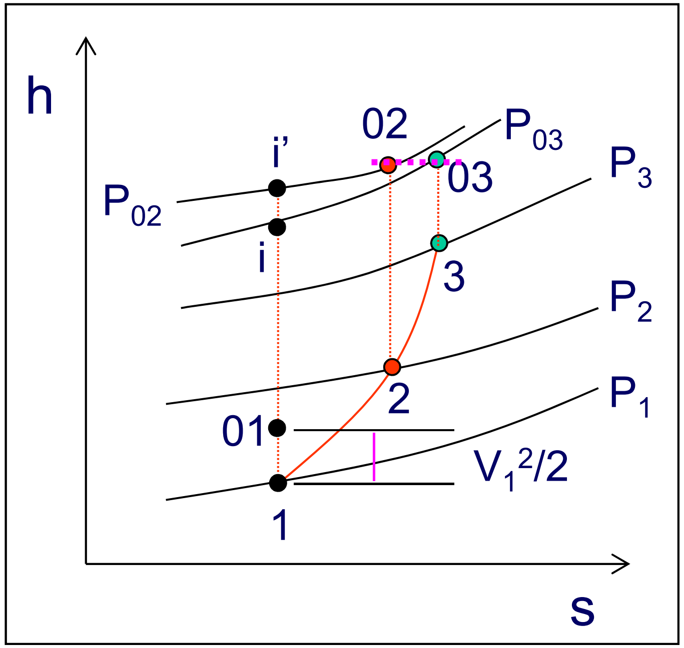
- Work of an ideal, isentropic, compression to the actual final pressure P_3 from 01 to i

$$E_i = C_p(T_i - T_{01}) = C_p T_{01}\left(\frac{T_i}{T_{01}} - 1\right)$$

$$= C_p T_{01}\left[\left(\frac{P_i}{P_{01}}\right)^{(\gamma-1)/\gamma} - 1\right]$$

$$= C_p T_{01}\left[\left(\frac{P_{03}}{P_{01}}\right)^{(\gamma-1)/\gamma} - 1\right]$$

eq3



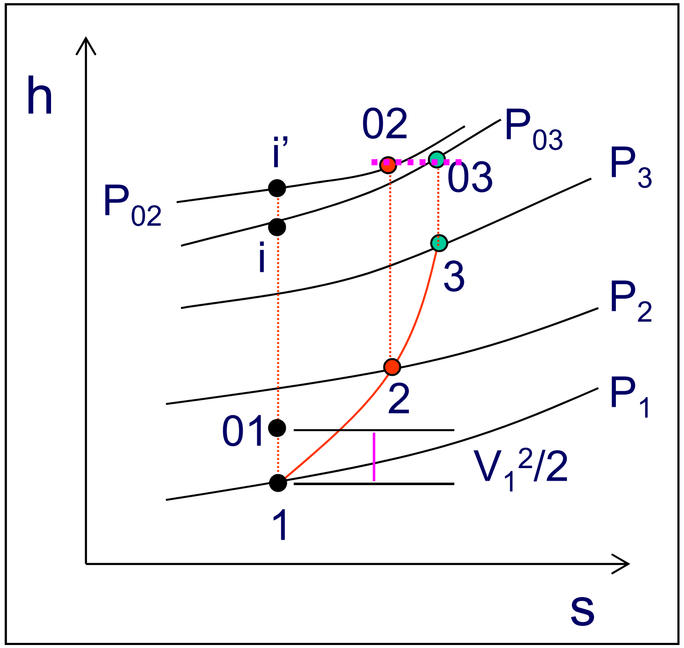
NextMEMS

Compressor Efficiency

- Compressor efficiency is defined as the ratio of the useful increase of fluid energy divided by the actual energy input to the fluid

$$\eta_c = \frac{E_i}{E} = \frac{T_i - T_{01}}{T_{03} - T_{01}}$$

eq4



NEXT MEMS

Overall Pressure Ratio

- Based on Eq. 2, 3 and 4, the overall pressure ratio is

$$\frac{P_{03}}{P_{01}} = \left(1 + \frac{U_2 V'_{u2} \eta_c}{c_P T_{01} \eta_m}\right)^{\gamma/(\gamma-1)}$$

η_c **experimentally determined quantity**

Stanitz equation

$$\mu_S = \frac{V'_{u2}}{V_{u2}} = 1 - \frac{0.63\pi}{n_B}\left(\frac{1}{1-\varphi_2 \cot\beta_2}\right)$$

useful in the range of $45^o < \beta_2 < 90^o$

Slip coefficient

$$\mu_s = \frac{V_{u2'}/U_2}{V_{u2}/U_2}$$

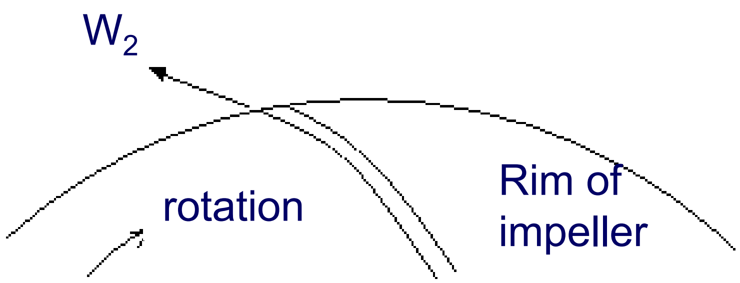
$$= 1 - \frac{\pi \sin\beta_2}{n_B}\left(\frac{U_2}{V_{u2}}\right)$$

* Pressure ratio = f(ideal velocity triangle at the impeller exit, the number of vanes, the inlet total temperature, the stage and mechanical efficiencies)

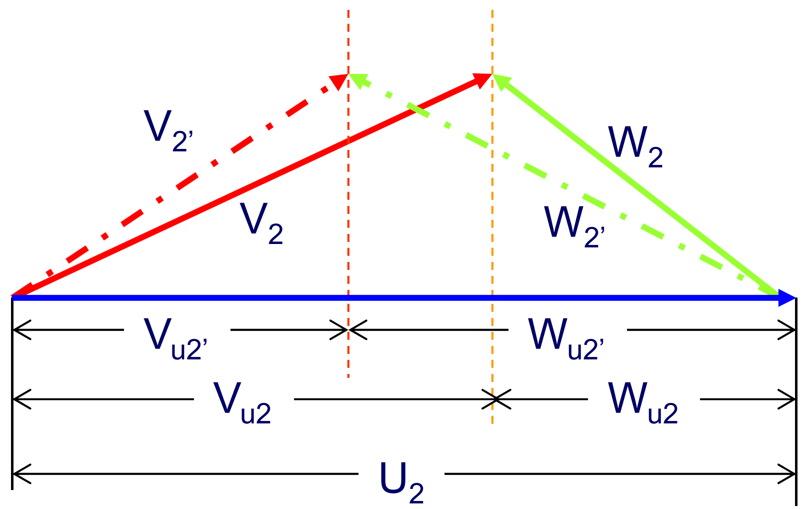
Design of Impeller

- The impeller is usually designed with a number of unshrouded blades for the inlet operating conditions,

$$N,\ \dot{m},\ P_{01}\ and\ T_{01}$$



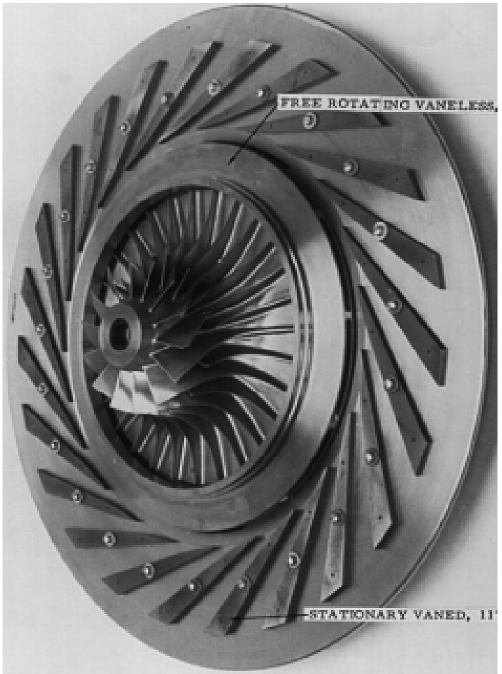
$\beta_2 < 90^o$ but bent near the leading edge to conform to the direction of the relative velocity W1 at the inlet

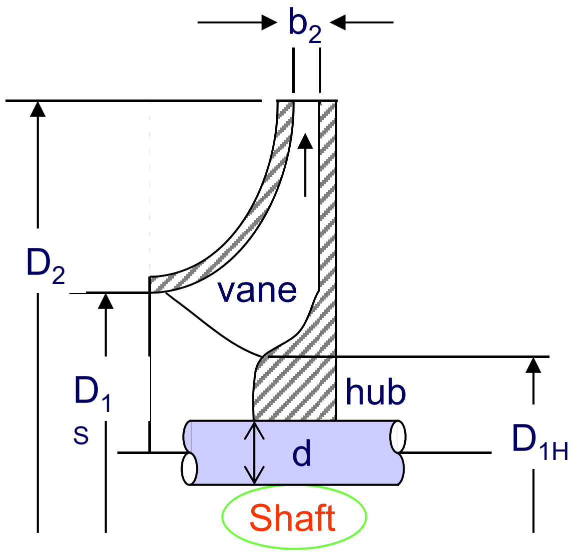


NEXTMEMS

Design of Impeller

- Determine the shaft diameter





Compute specific speed → **Determine shaft torque** → **Determine shaft diameter**

Determine efficiency
$\eta = f(Ns, Q)$

$T = P/N$

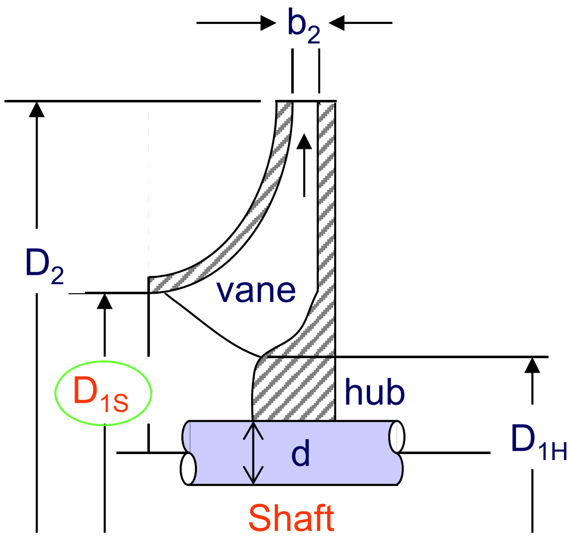
$P = E/\eta_m$

Stress-strain relation

NextMEMS

Design of Impeller

- Equations for the impeller inlet is completed by velocity triangle and gas property relations
  - Determine the shroud diameter



For a perfect gas

$$Pv = P/\rho = RT$$

For the isentropic process

$$Pv^{\gamma} = P/\rho^{\gamma} = Const.$$

$$\frac{T_{o1}}{T_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)$$

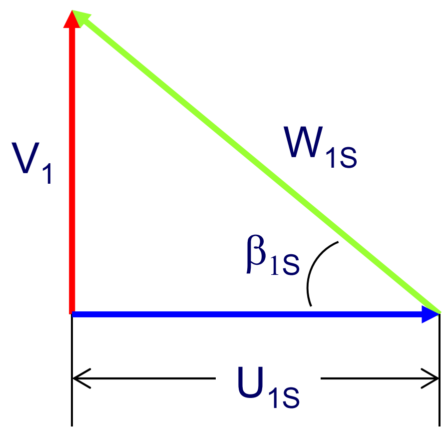
$$\frac{P_{o1}}{P_1} = \left(\frac{T_{o1}}{T_1}\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{P_{o1}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma - 1}}$$

Refer : http://www.grc.nasa.gov/WWW/K-12/airplane/isentrop.html

Design of Impeller

- At the shroud inlet with velocity triangle and gas property relations



$$W_1 = \sqrt{\left(V_1^2 + U_{1S}^2\right)}$$

For the inlet operating conditions,

$$N,\ \dot{m},\ P_{01}\ and\ T_{01}$$

The relative Mach number has its minimum where β_{1S} is approximately 32° (Shepherd, 1956)

Choose inlet relative Mach number

$$W_{1S} = M_{R1S}\, a_1\,, \qquad a_1 = \left(\gamma R T_1\right)^{1/2}$$

$$T_1 = \frac{T_{01}}{1 + (\gamma - 1) M_1^2 / 2}, \qquad M_1 = \frac{V_1}{a_1}$$

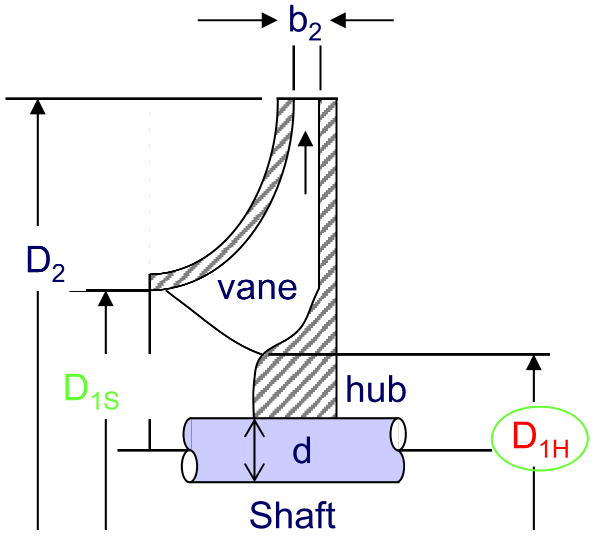
$$V_1 = W_{1S} \sin 32^o,\ U_{1S} = W_{1S} \cos 32^o$$

Shroud diameter $\qquad D_{1S} = \dfrac{2U_{1S}}{N}$

NEXTMEMS

Design of Impeller

- Determine the hub diameter & the fluid angle at the hub



From the mass flow rate

$$\dot{m} = \rho_1 A_{1S} V_1 - \rho_1 A_{1H} V_1$$

$$= \rho_1 \pi \frac{D_{1S}^2}{4} V_1 - \rho_1 \pi \frac{D_{1H}^2}{4} V_1$$

Hub diameter $D_{1H} = \left(D_{1S}^2 - \frac{4\dot{m}}{\rho_1 \pi V_1} \right)^{1/2}$

$$\rho_1 = \frac{P_1}{RT_1} \quad \text{for ideal gas}$$

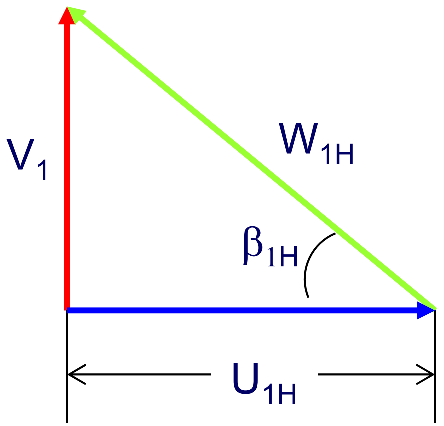
$$P_1 = P_{o1} \left(1 + \frac{\gamma - 1}{2} M_1^2 \right)^{-\frac{\gamma}{\gamma - 1}}$$

$$T_1 = T_{o1} \left(1 + \frac{\gamma - 1}{2} M_1^2 \right)^{-1}$$

NEXT MEMS

Design of Impeller

- Determine the hub diameter & the fluid angle at the hub



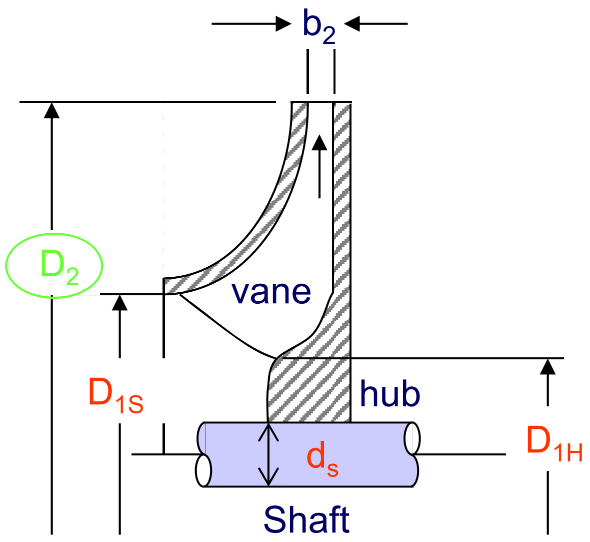
Hub angle

$$\beta_{1H} = \tan^{-1}\left(\frac{V_1}{U_{1H}}\right)$$

$$U_{1H} = \frac{ND_{1H}}{2}$$

Design of Impeller

- Determine the impeller diameter



Dimensional specific speed

$$N_s = \frac{NQ_1^{1/2}}{H^{3/4}}$$

Calculate output head

$$H = \frac{E_i}{g}$$ Ideal input energy

$$E_i = c_p\left(T_{03} - T_{01}\right)$$

$$= c_p T_{01}\left[\left(\frac{P_{03}}{P_{01}}\right)^{(\gamma-1)/\gamma} - 1\right]$$

given

NEXTMEMS

Design of Impeller

$$N_s = \frac{\varphi^{1/2}}{\psi^{3/4}} = \frac{NQ^{1/2}}{(gH)^{3/4}}$$

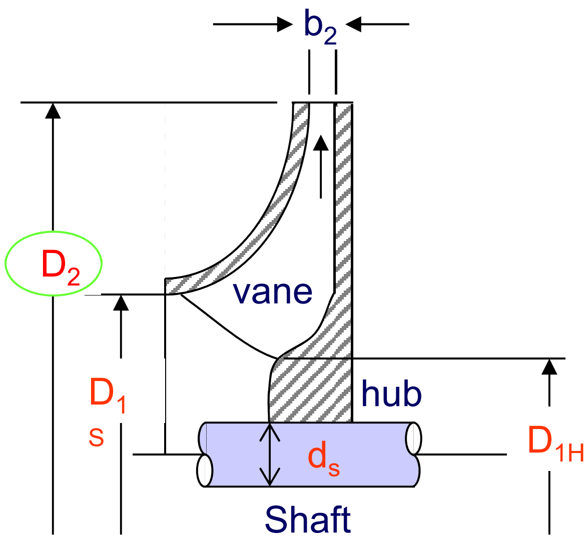
Choose compressor efficiency

Dimensional specific speed

	Compressor efficiency η_c				
	0.40	0.50	0.60	0.70	0.80
50	2.42	2.65	2.91	–	–
60	1.94	2.14	2.26	–	–
65	1.77	1.92	2.02	2.14	–
70	1.66	1.82	1.89	1.96	–
80	1.44	1.55	1.63	1.68	–
85	1.36	1.48	1.53	1.57	1.70
90	1.30	1.39	1.43	1.46	1.59
100	1.16	1.25	1.29	1.32	1.41
110	1.07	1.14	1.17	1.21	1.29
120	1.00	1.06	1.10	1.15	1.22
130	0.91	1.00	1.03	1.08	1.18
140	0.87	0.96	1.00	1.06	–
150	0.83	0.94	1.00	1.07	
160	0.80	0.91	1.00	1.04	
170	0.80	0.91	1.00	1.11	
180	0.80	0.91	1.00	–	
190	0.79	0.91	1.01	–	–
200	0.79	0.91	–	–	–

Dimensional specific diameter

Source: Scheel, L. F. 1972. *Gas Machinery*. Gulf Publishing Co., Houston.



Read highest compressor efficiency and corresponding specific diameter

$$D_s = \frac{\psi^{1/4}}{\varphi^{1/2}} = \frac{D(gH)^{1/4}}{Q^{1/2}}$$

NEXTMEMS

Design of Impeller

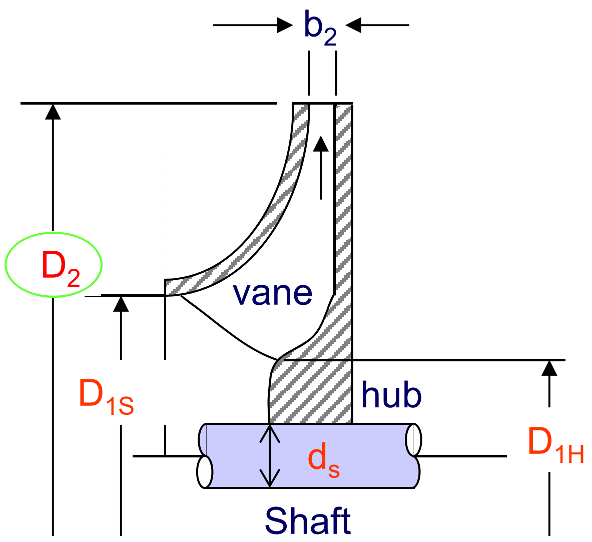
$$N_s = \frac{\psi^{1/4}}{\varphi^{1/2}} = \frac{NQ_1^{1/2}}{H^{3/4}}$$

$$D_s = \frac{\psi^{1/4}}{\varphi^{1/2}} = \frac{D_2(gH)^{1/4}}{Q^{1/2}}$$

$$Q_1 = \frac{\dot{m}}{\rho_1}, \quad \rho_1 = \frac{P_1}{RT_1}$$

$$H = \frac{E_i}{g}, \quad E_i = c_p\left(T_{03} - T_{01}\right)$$

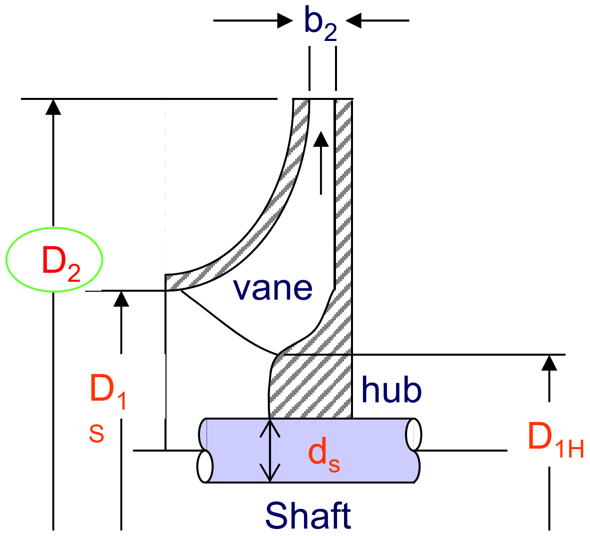
$$= c_p T_{01}\left[\left(\frac{P_{03}}{P_{01}}\right)^{(\gamma-1)/\gamma} - 1\right]$$



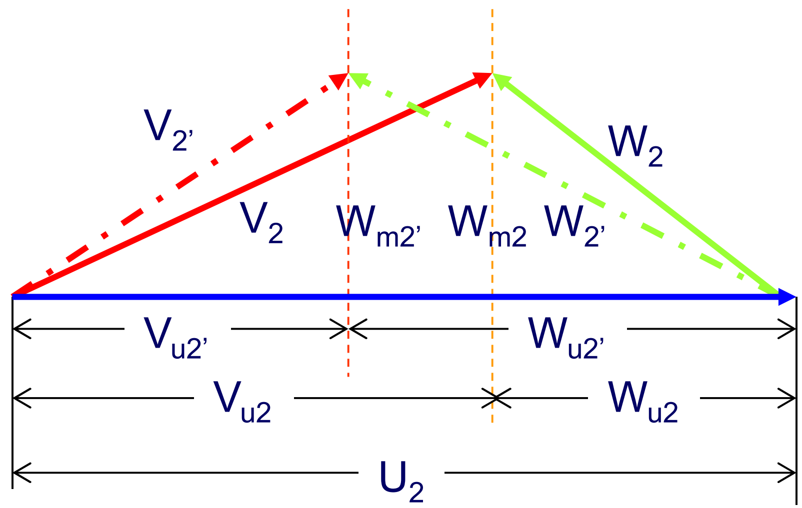
NEXTMEMS

Design of Impeller

- Determine the impeller vane angle and number of vanes



Velocity triangle at impeller



Impeller vane angle

$$\beta_2 = \tan^{-1}\left(\frac{W_{m2}}{W_{u2}}\right)$$

Design of Impeller

Calculate the actual tangential velocity

assume the slip coefficient of 0.85~0.90

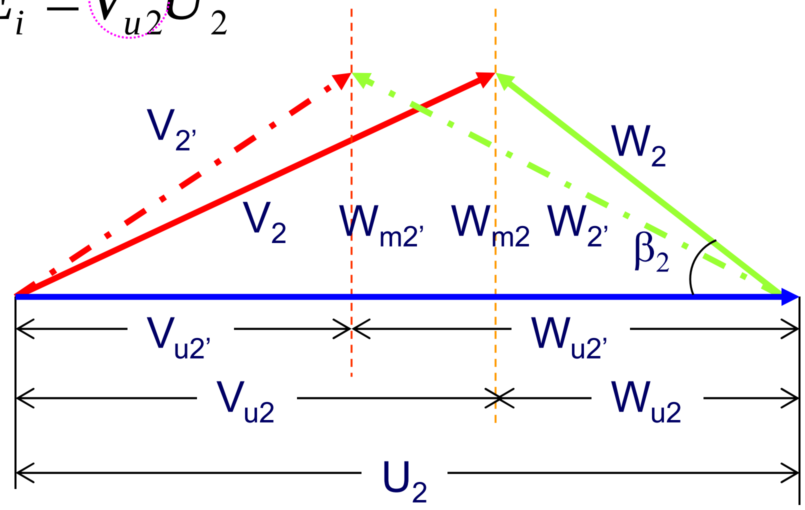
$$\mu_s = \frac{V_{u2'}}{V_{u2}} \rightarrow V_{u2} = \frac{V_{u2'}}{\mu_s} \leftarrow E = \frac{\eta_m}{\eta_c} E_i = V_{u2} U_2$$

Based on vector relation

$$W_{u2} = U_2 - V_{u2}$$

$$W_{m2} = \varphi_2 U_2$$

select a flow coefficient,
0.23~0.35



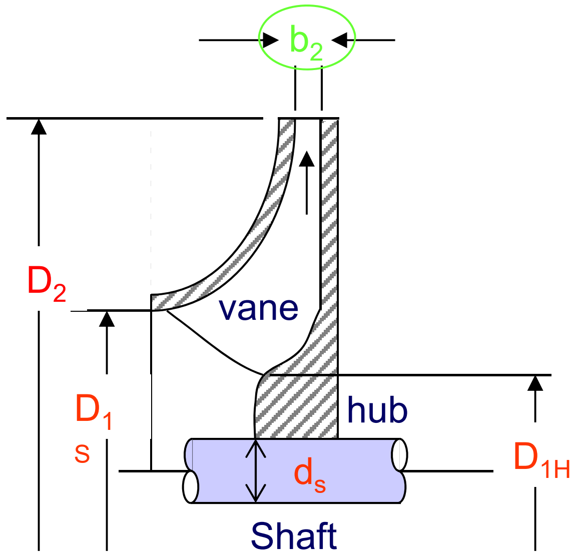
Design of Impeller

Number of vanes

$$\mu_S = \frac{V_{u2'}}{V_{u2}} = 1 - \frac{0.63\pi}{n_B}\left(\frac{1}{1-\varphi_2 \cot \beta_2}\right)$$

Design of Impeller

- Determine the impeller vane tip thickness



Impeller tip thickness

$$b_2 = \frac{\dot{m}}{\rho_2 2\pi r_2 W_{m2}} = \frac{\dot{m}}{\rho_2 \pi D_2 W_{m2}}$$

$$\rho_2 = \frac{P_2}{RT_2}$$

Design of Impeller

Estimate impeller efficiency

$$\chi = \frac{1-\eta_I}{1-\eta_c}$$

assume 0.5~0.6

: ratio of impeller losses to compressor losses

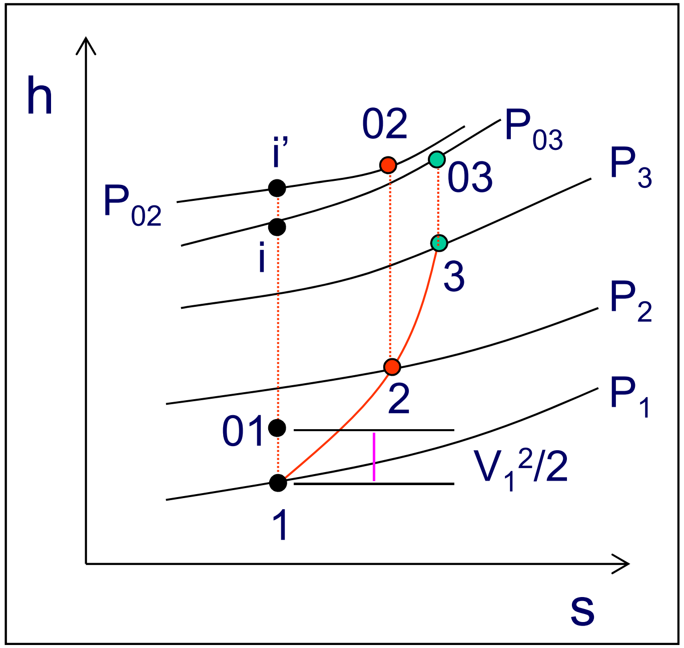
$$\eta_I = \frac{T_{i'} - T_{01}}{T_{02} - T_{01}}$$

$$E = \eta_m (h_{03} - h_{01}) = \eta_m c_p (T_{03} - T_{01})$$

$$\rightarrow T_{03} = T_{01} + \frac{E}{\eta_m c_p} = T_{02}$$

$$T_{02} - T_{01} = \frac{E}{\eta_m c_p}$$

enthalpy-entropy diagram



Assmp. :
no external work and heat transfer
= h_{02}= h_{03}, T_{02}= T_{03}

NEXTMEMS

Design of Impeller

From isentropic process

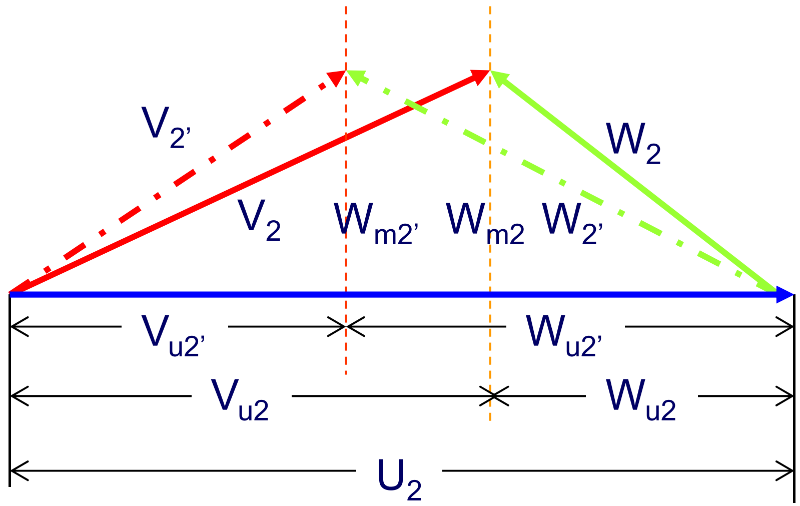
$$\frac{P_{i'}}{P_{01}} = \left(\frac{T_{i'}}{T_{01}}\right)^{\gamma/(\gamma-1)} = \frac{P_{02}}{P_{01}}$$

$$\frac{P_{o2}}{P_2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_2 = T_{02} - \frac{V_{2'}^2}{2c_p}$$

$$V_{2'}^2 = W_{m2}^2 + V_{u2'}^2$$

where $W_{m2} = W_{m2'}$



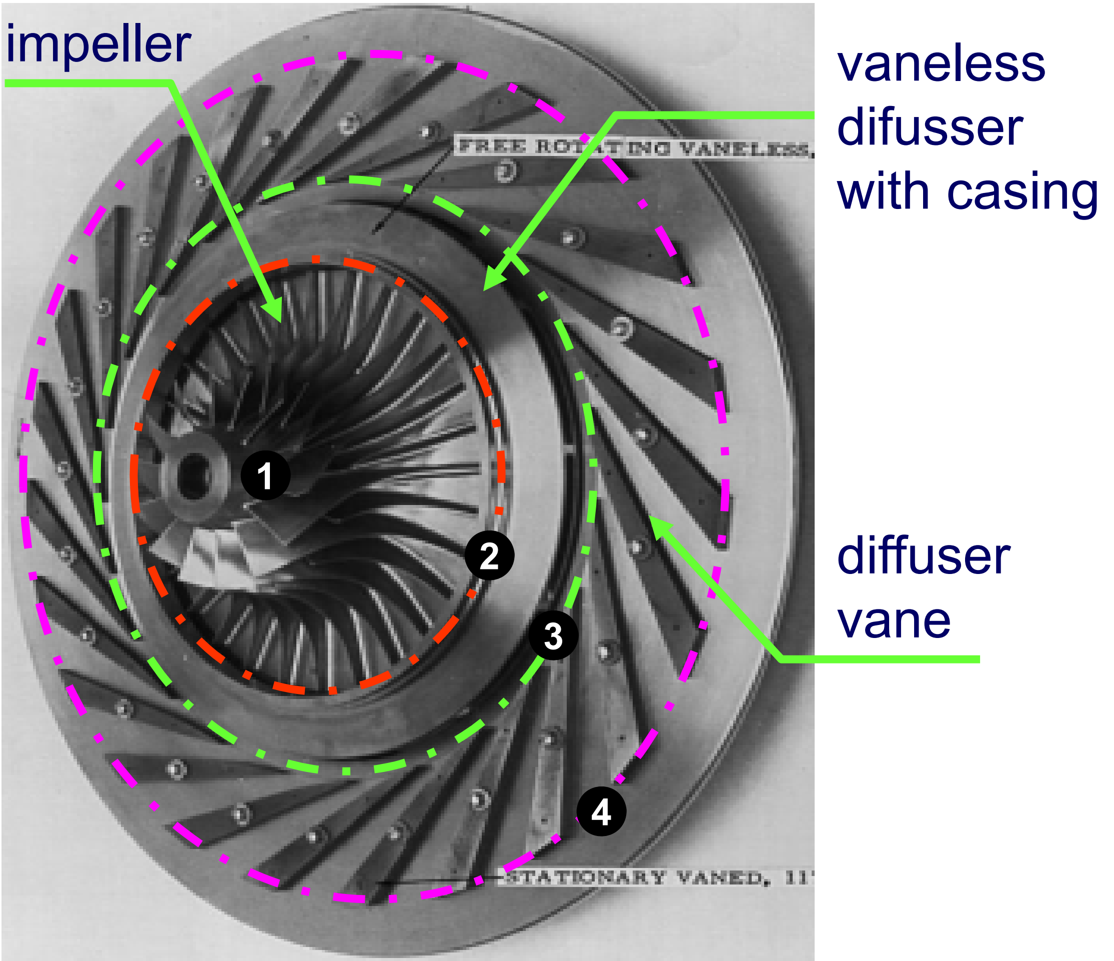
$$\rho_2 = \frac{P_2}{RT_2}$$

Impeller tip thickness

$$b_2 = \frac{\dot{m}}{\rho_2 2\pi r_2 W_{m2}} = \frac{\dot{m}}{\rho_2 \pi D_2 W_{m2}}$$

NextMEMS

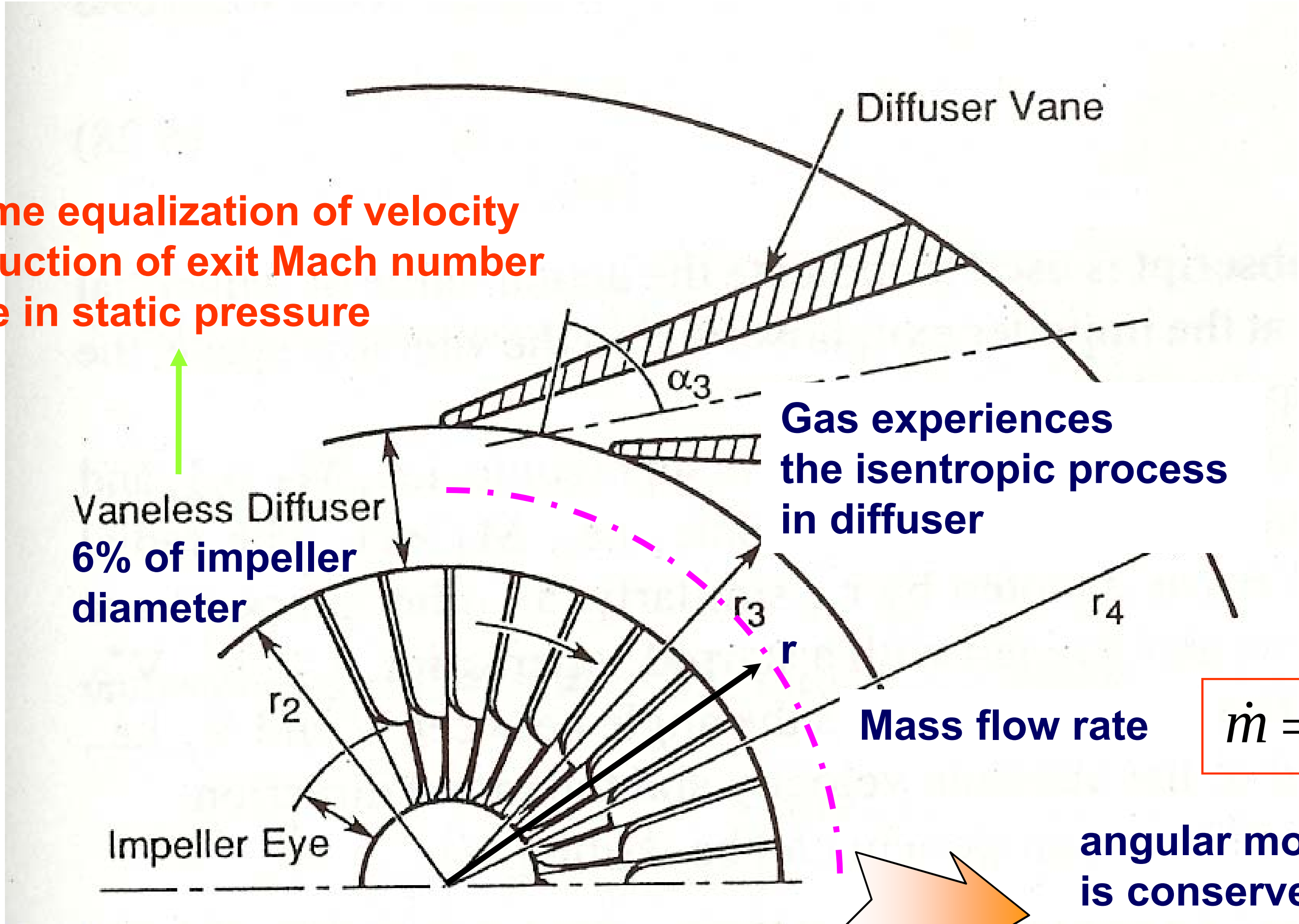
Diffuser Composition



NEXT MEMS

Diffuser Design

- some equalization of velocity
- reduction of exit Mach number
- rise in static pressure



6% of impeller diameter

Gas experiences the isentropic process in diffuser

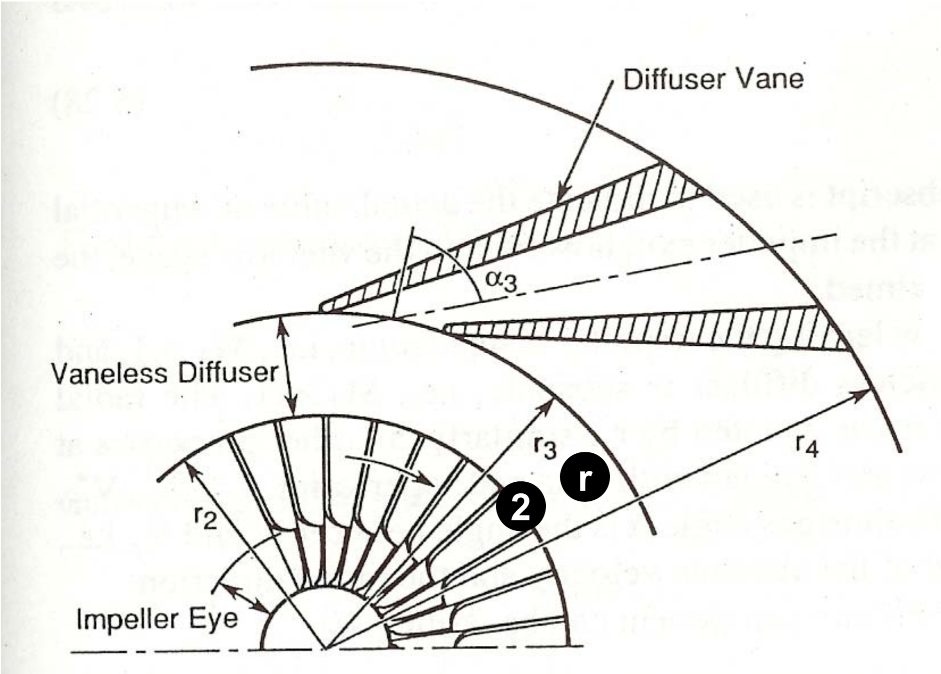
Mass flow rate $\dot{m} = 2\pi\rho b r V_m$

angular momentum is conserved in diffuser

$$rV_u = const$$

NEXTMEMS

Vaneless Diffuser Design



From mass flow rate

$$\dot{m} = 2\pi\rho b r V_m$$

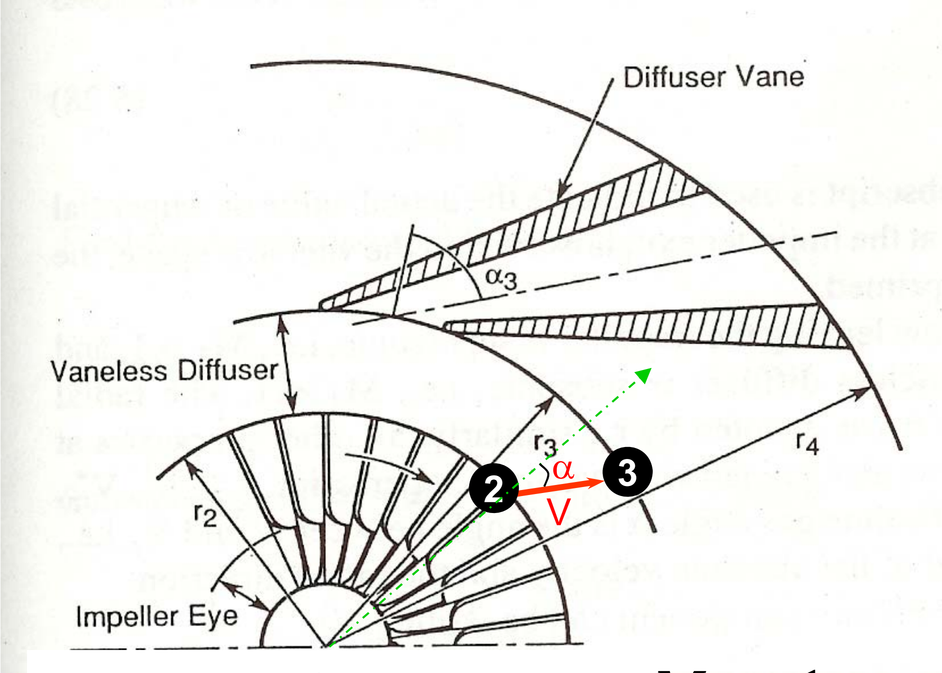
For constant diffuser width, b

$$\rho_2 r_2 V_{2m} = \rho r V_m$$

Angular momentum is conserved in the vaneless space

$$r_2 V_{u2'} = r V_u$$

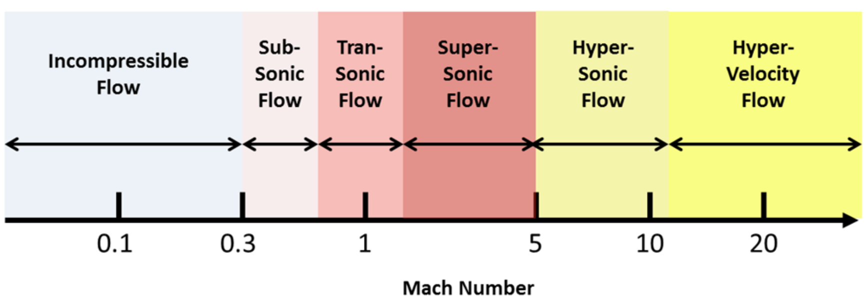
Vaneless Diffuser Design



at position 2: supersonic $M_{2'} > 1$

at position 3: subsonic $M_3 < 1$

Mach Number Flow Regimes



All properties are denoted at the plane of sonic flow, M = 1

$$\rho^* \quad r^* \; T^* \; V_m^* \; \alpha^*$$

For constant diffuser width, b

$$V_r = V_m = V\cos\alpha$$

From the continuity equation

$$\rho r V\cos\alpha = \rho^* r^* V^* \cos\alpha^* \quad \text{---}(1)$$

Angular momentum conservation

$$rV\sin\alpha = r^* V^* \sin\alpha^* \quad \text{---}(2)$$

NEXTMEMS

Vaneless Diffuser Design

Dividing Eq. 2 by Eq. 1,

$$\frac{\tan\alpha}{\rho} = \frac{\tan\alpha^*}{\rho^*}$$

Assuming an isentropic flow in the vaneless region

$$\frac{T}{T^*} = \left(\frac{\rho}{\rho^*}\right)^{\gamma-1}$$

$$T = \frac{T_o}{1+\frac{(\gamma-1)}{2}M^2}$$

For M=1, T=T* $\quad T^* = \frac{2T_o}{\gamma+1}$

For the isentropic process

$$Pv^\gamma = P/\rho^\gamma = Const.$$

$$\frac{T_{o1}}{T_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)$$

$$\frac{P_{o1}}{P_1} = \left(\frac{T_{o1}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{o1}}{P_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

NextMEMS

Vaneless Diffuser Design

Assuming an isentropic flow in the vaneless region

$$\frac{\rho}{\rho^*} = \left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{1/(\gamma-1)}$$

$$\tan\alpha = \tan\alpha^* \frac{\rho}{\rho^*}$$

$$= \tan\alpha^* \left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{1/(\gamma-1)}$$

α^* can be evaluated by substituting $\alpha = \alpha_{2'}$ and $M = M_{2'}$

For the isentropic process

$$\frac{T_{o1}}{T_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)$$

$$\frac{T}{T^*} = \left(\frac{\rho}{\rho^*}\right)^{\gamma-1}$$

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

NextMEMS

Vaneless Diffuser Design

From angular momentum conservation

$$\frac{r^* \sin\alpha^*}{r\sin\alpha} = \frac{V}{V^*} = \frac{V}{a}\frac{a}{a^*} \qquad \leftarrow a = (\gamma RT)^{1/2}$$

$$= M\left(\frac{T}{T^*}\right)^{1/2} \leftarrow \frac{T}{T^*} = \left(\frac{\rho}{\rho^*}\right)^{\gamma-1} \leftarrow \frac{\rho}{\rho^*} = \left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{1/(\gamma-1)}$$

$$\frac{r^* \sin\alpha^*}{r\sin\alpha} = M\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{-1/2}$$

The radial position r* can be found by substituting $r=r_2$ and $M = M_2$,

NextMEMS

Vaneless Diffuser Design

Select M_3

α_3 can be evaluated from a known M_3

$$\tan\alpha = \tan\alpha^* \left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{1/(\gamma-1)} \quad \leftarrow \frac{\tan\alpha}{\rho} = \frac{\tan\alpha^*}{\rho^*}$$

r_3 can be evaluated from the known α_3 and M_3

$$\frac{r^*\sin\alpha^*}{r\sin\alpha} = M\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{-1/2}$$

Thank you for attention

NEXTMEMS